Overview

- Introduction (Fox-Ch1)
  - Response function
  - Optical processes
  - Optical constants
- Waves in solids (Fox-Appendix A)
  - Maxwell equations and wave equation
- Models (Fox-Ch2,3,7)
  - Lorentz model
  - Drude-Lorentz model
  - Transition rates, QM treatment
- Magneto-optical effects, XMCD
- Inelastic light scattering
- Non-linear optics
- Time resolved optics
- Optical modification of matter

Absorption

Traveling wave will be damped

$$E(x,t) = E_0 \exp(i(qx-\omega t))$$
$$= E_0 \exp(i(q_r x-\omega t))*\exp(-q_c\, x)$$

$$I(x)=I_0 \exp(-2q_c x)$$

Absorption coefficient

$$\alpha = 2q_c = 2\, n_c\, \omega/c$$

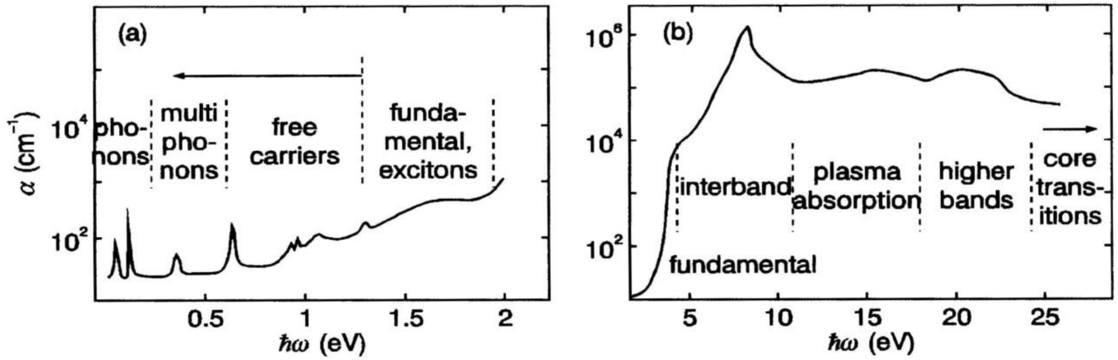
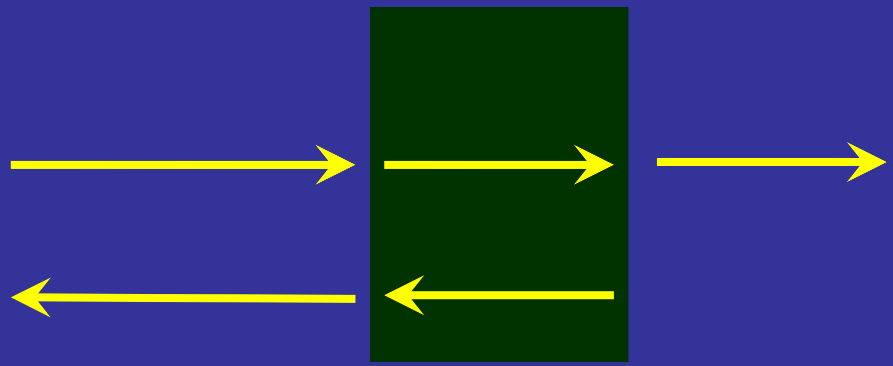


Fig. 6.2. Schematic representation of the absorption coefficient in solids for the various excitation processes; low-energy range (**a**), high-energy range (**b**).

Transmission

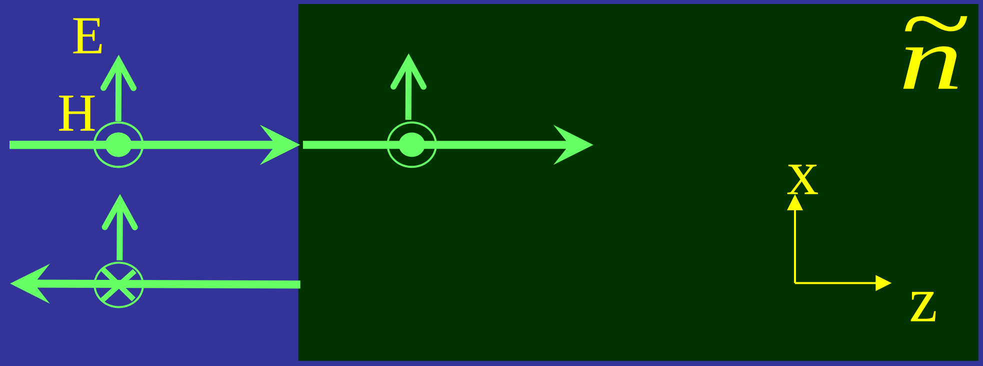


$$T = (1 - R_1)\, e^{-\alpha L} (1 - R_2) = (1 - R)^2\, e^{-\alpha L}$$

Skin depth: $\delta = \dfrac{2}{\alpha}$ (field penetration)

Penetration depth: $d = \dfrac{1}{\alpha}$ (flux penetration)

Reflection



At boundary: $\Delta E_T = 0$

$\Delta H_T = 0$

$$r = \frac{E_x^r}{E_x^i} = \frac{\tilde{n} - 1}{\tilde{n} + 1}$$

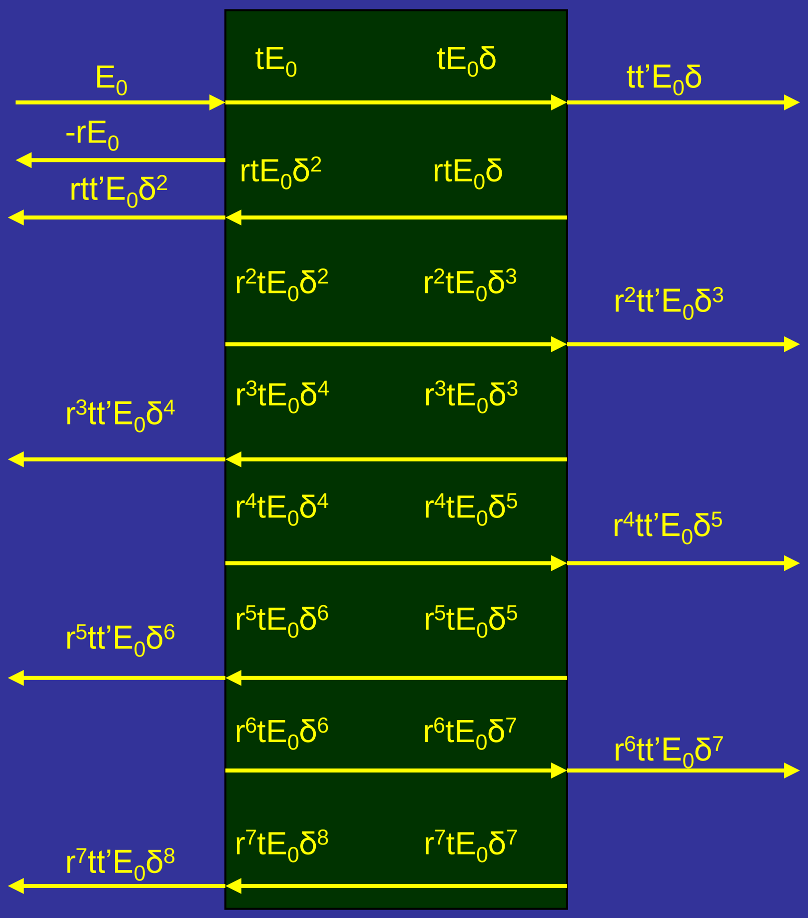
$$E_x^i + E_x^r = E_x^t$$

$$H_y^i - H_y^r = H_y^t \quad \Rightarrow \quad E_x^i - E_x^r = \tilde{n} E_x^t$$

$$R = |r|^2 = rr^* = \left|\frac{\tilde{n} - 1}{\tilde{n} + 1}\right|^2$$

$$T = 1 - R$$

Interference: Fabry-Perot etalon



$$T = \frac{1}{1 + F(R)\sin^2(\Delta/2)}$$ (Airy function)

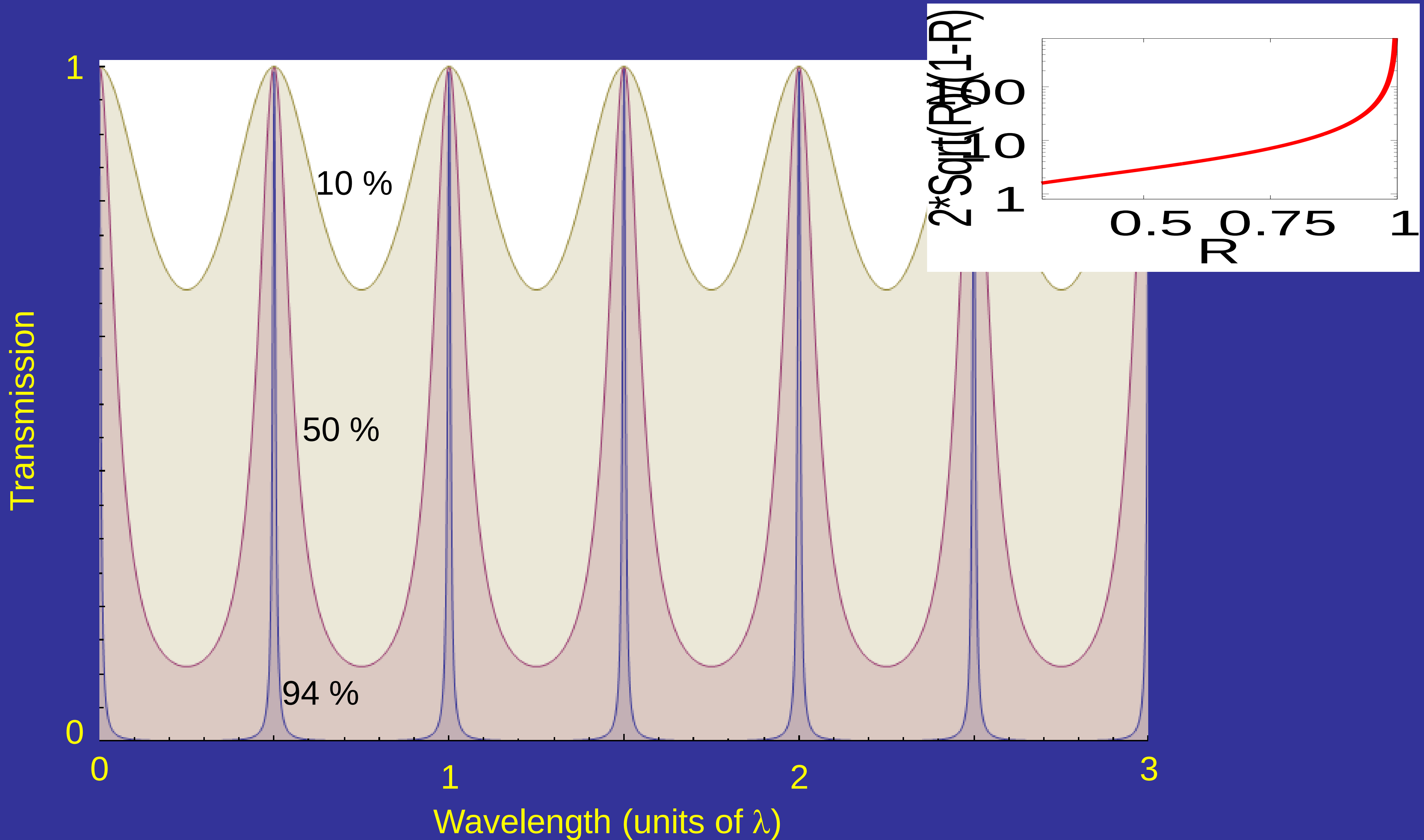
$$F(R) = \frac{4R}{(1-R)^2}$$ (finesse)

$$\delta = e^{-i\cdot\Delta/2}; \qquad \Delta = 2\cdot k\cdot d = 4\pi\cdot\frac{d}{\lambda_0/n}$$

Resolution $\Delta\lambda / \lambda \sim 2R^{1/2}/(1-R)$

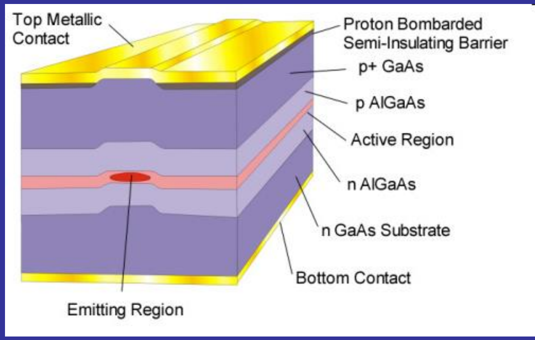
See also notes on web

Fabry Perot Transmission

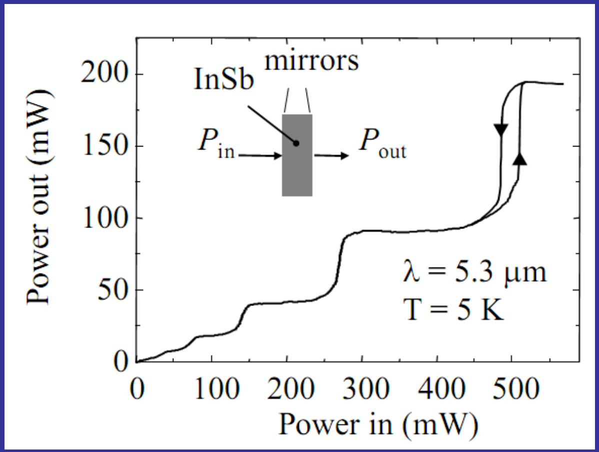


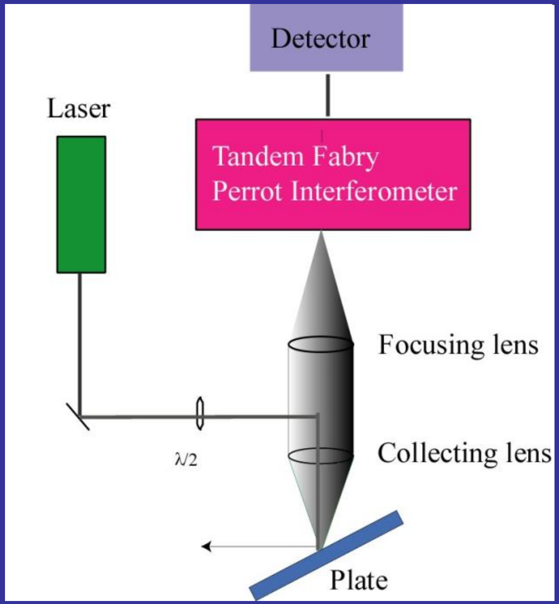
Uses of FP etalon

- Energy selective filter (Bandpass filter → Brillouin scattering, Single mode lasers)
- Diode Laser resonator (FOX par. 5.4.3)
- High density non-linear effects (FOX par 11.2.2; 11.4.3)



Intensity dependent refractive index
Bistability due to phase shift in cavity





$$T = \frac{1}{1 + F(R)\sin^2(\Delta/2)}$$

$$\text{When} \quad \Delta = \frac{4\pi nd}{\lambda} = 2\pi \times j \quad \text{then T} = 100\% \ (j : \text{integer})$$

$$\frac{2(n_0 + I\, n_2)d}{\lambda} = \text{integer}$$

Maxwell equations in matter

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{P} = \varepsilon_0 \overleftrightarrow{\chi}_e \vec{E}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \overleftrightarrow{\varepsilon}_r \vec{E}$$

$$\vec{M} = \overleftrightarrow{\chi}_m \vec{H}$$

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right) = \mu_0 \overleftrightarrow{\mu}_r \vec{H}$$

$$\vec{j} = \overleftrightarrow{\sigma} \vec{E}$$

Constitutive equations & Maxwell Eq.

$$\vec{P} = \varepsilon_0 \overleftrightarrow{\chi}_e \vec{E}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \overleftrightarrow{\varepsilon}_r \vec{E}$$

$$\vec{M} = \overleftrightarrow{\chi}_m \vec{H}$$

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right) = \mu_0 \overleftrightarrow{\mu}_r \vec{H}$$

$$\vec{j} = \overleftrightarrow{\sigma} \vec{E}$$

assume μ_r *scalar*

$$(1) \qquad \nabla \cdot \varepsilon_0 \overleftrightarrow{\varepsilon}_r \vec{E} = \rho$$

$$(2) \qquad \nabla \cdot \vec{H} = 0$$

$$(3) \qquad \nabla \times \vec{E} = -\mu_0 \mu_r \frac{\partial \vec{H}}{\partial t}$$

$$(4) \qquad \nabla \times \vec{H} = \overleftrightarrow{\sigma} \vec{E} + \varepsilon_0 \overleftrightarrow{\varepsilon}_r \frac{\partial \vec{E}}{\partial t}$$

Wave equation (isotropic media)

$$(1)\quad \nabla\cdot\varepsilon_0\overleftrightarrow{\varepsilon}_r\vec{E}=\rho$$

$$(2)\quad \nabla\cdot\vec{H}=0$$

$$(3)\quad \nabla\times\vec{E}=-\mu_0\mu_r\frac{\partial\vec{H}}{\partial t}$$

$$(4)\quad \nabla\times\vec{H}=\overleftrightarrow{\sigma}\vec{E}+\varepsilon_0\overleftrightarrow{\varepsilon}_r\frac{\partial\vec{E}}{\partial t}$$

From equation (3), assuming real scalar response fncts & μ_r=1

$$\nabla\times\left(\nabla\times\vec{E}\right)=\nabla\left(\nabla\cdot\vec{E}\right)-\nabla^2\vec{E}\underset{\rho=0}{=}-\nabla^2\vec{E}$$

$$\text{So:}\quad \nabla^2\vec{E}=\mu_0\frac{\partial\nabla\times\vec{H}}{\partial t}$$

Using eq. (4) this leads to a damped harmonic wave equation

$$\nabla^2\vec{E}=\mu_0\left[\varepsilon_0\varepsilon_r\frac{\partial^2\vec{E}}{\partial t^2}+\sigma\frac{\partial\vec{E}}{\partial t}\right]$$

Wave equation

Harmonic solutions of the form $e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ and assuming scalar response functions

$$-k^2\vec{E}=-\mu_0\left(\varepsilon_0\varepsilon_r\omega^2+i\omega\sigma\right)\vec{E}$$

$$k^2=\mu_0\varepsilon_0\varepsilon_r\omega^2+i\omega\mu_0\sigma$$

Dispersion

Response: Dielectric function ε

Velocity of light: v = c/n

Refractive index:
n is a complex function

$$\tilde{n} = n + i\kappa = n_r + i n_c$$

$$\tilde{n}^2 = \varepsilon$$

$$\varepsilon_r = n_r^2 - n_c^2 \qquad n_r = \frac{1}{\sqrt{2}}\left(\varepsilon_1 + \sqrt{\varepsilon_1 + \varepsilon_2}\right)^{1/2}$$

$$\varepsilon_c = 2 n_r n_c \qquad n_c = \frac{1}{\sqrt{2}}\left(-\varepsilon_1 + \sqrt{\varepsilon_1 + \varepsilon_2}\right)^{1/2}$$

Wave vector q is then also complex

$$q = \frac{2\pi}{\lambda} = \frac{2\pi}{\lambda_0 / \tilde{n}} = \tilde{n}\frac{\omega}{c} = q_r + i q_c$$

$$q_r = n_r \frac{\omega}{c}; \quad q_c = n_c \frac{\omega}{c}$$

Complex response

$$k^2 = \mu_0 \varepsilon_0 \varepsilon_r \omega^2 + i\omega\mu_0\sigma$$

$$k \equiv n\frac{\omega}{c} \quad \Rightarrow \quad k^2 = n^2\frac{\omega^2}{c^2}; \quad c^2 = \frac{1}{\varepsilon_0\mu_0}$$

$$n^2 = \varepsilon_1 + i\frac{\sigma}{\varepsilon_0\omega} \equiv \hat{\varepsilon}_r$$ Complex dielectric function

Other way around: complex optical conductivity:

$$j = \frac{dP}{dt} = \varepsilon_0\chi\frac{dE}{dt} = -i\omega\varepsilon_0\hat{\chi}E \equiv \hat{\sigma}E$$

$$\hat{\sigma} = -i\omega\varepsilon_0\hat{\chi} = -i\omega\varepsilon_0(1-\hat{\varepsilon}_r)$$

Optical functions

Wave vector $k = \frac{\omega}{c} n = \frac{2\pi n}{\lambda_{vac}}$

Dielectric function $\varepsilon_r = n^2$

Optical conductivity $\sigma = -i\omega\varepsilon_0(\varepsilon_r - 1)$

Dielectric susceptibility $\chi = \varepsilon_r - 1$

Absorption coefficient $\alpha = 2k_2 = \frac{4\pi n_2}{\lambda_{vac}}$

Skin depth $\delta = \frac{2}{\alpha}$

$$n_1 = \frac{\sqrt{|\varepsilon_r| + \varepsilon_1}}{\sqrt{2}}; \quad n_2 = \frac{\sqrt{|\varepsilon_r| - \varepsilon_1}}{\sqrt{2}}$$

$$\varepsilon_1 = n_1{}^2 - n_2{}^2$$

$$\varepsilon_2 = 2n_1 n_2$$

$$\sigma_1 = \omega\varepsilon_0\varepsilon_2$$

$$\sigma_2 = -\omega\varepsilon_0(\varepsilon_1 - 1)$$

Reflection coefficient (perp. Incidence)

$$R_\perp = \frac{|n-1|}{|n+1|}$$

General solution

The solutions of the Maxwell eqns. are generally plane waves of the form

$$\bar{A} = \bar{A}_0 e^{i\omega\left(\frac{n}{c}\bar{r}\cdot\bar{s}-t\right)},$$

with n the refractive index, c the speed of light, and $\bar{s}$ the propagation direction. The equations $\bar{\nabla} \times \bar{E} = -\partial\bar{B}/\partial t$ and $\bar{\nabla} \times \bar{H} = \partial\bar{D}/\partial t$ (no free current) lead to:

$$\begin{aligned} i\omega\frac{n}{c}\bar{s} \times \bar{H} &= -i\omega\bar{D} \\ i\omega\frac{n}{c}\bar{s} \times \bar{E} &= i\omega\bar{B} \end{aligned}$$

Or, together with the constitutive equation $\bar{B} = \mu_0\bar{H}$

$$\bar{D} = n^2\left[\bar{E} - (\bar{E}\cdot\bar{s})\bar{s}\right]$$

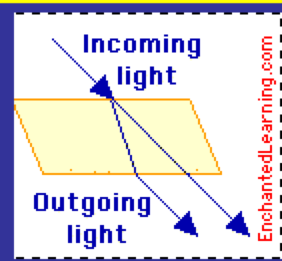
With the const. eq. $\bar{D} = \bar{\bar{\epsilon}}\bar{E}$ this gives

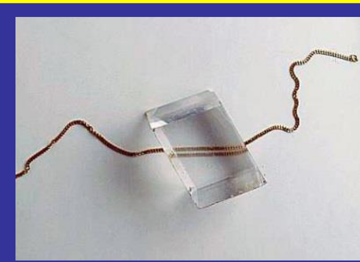
$$\boxed{\left(\bar{\bar{\epsilon}} - n^2\bar{\bar{I}}\right)\bar{E} = -n^2(\bar{E}\cdot\bar{s})\bar{s},}$$

which solutions (4, 2 for $+s$ and 2 for $-s$) give the electromagnetic propagation modes.

Example: Magneto optic Media

Linear birefringence: see FOX CH 1



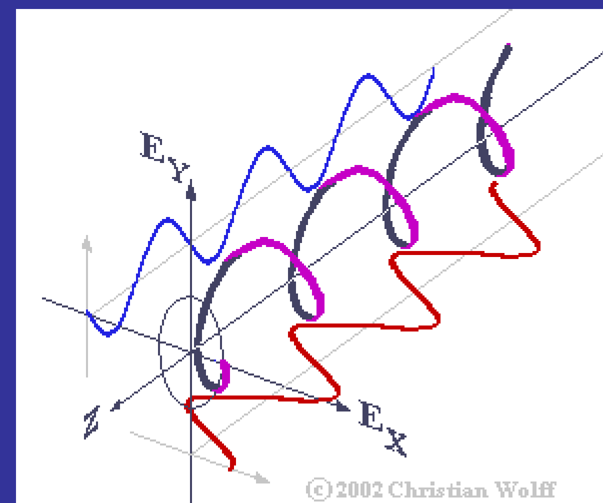


Presence of a magnetic field leads to imaginary off diagonal matrix elements

Assume wave propagating in z-direction, B-field in z-direction, E-field in xy plane

$$\overleftrightarrow{\varepsilon} = \begin{pmatrix} 1 & ia \\ -ia & 1 \end{pmatrix}$$

$$\left(\overleftrightarrow{\varepsilon} - n^2 \overleftrightarrow{I}\right)\vec{E} = 0 \quad \Rightarrow \quad n^2 = 1 \pm a$$



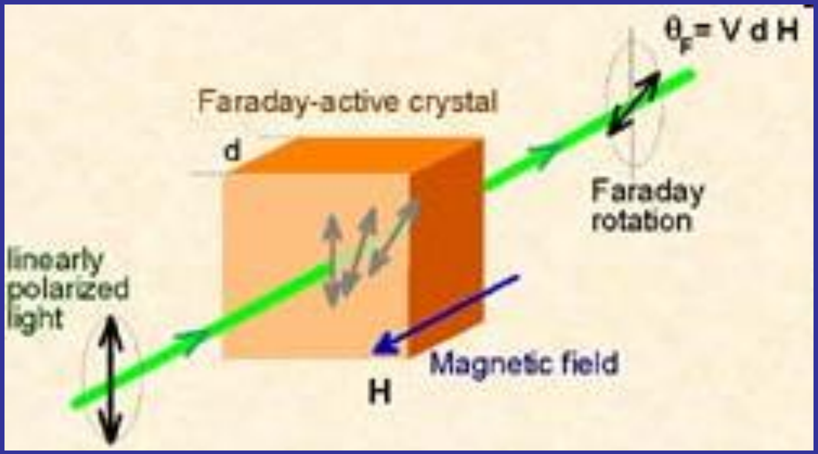
Eigenvectors: $\begin{pmatrix} i \\ 1 \end{pmatrix}, \begin{pmatrix} -i \\ 1 \end{pmatrix}$ or $\begin{pmatrix} e^{i\pi/2} \\ 1 \end{pmatrix}, \begin{pmatrix} e^{-i\pi/2} \\ 1 \end{pmatrix}$

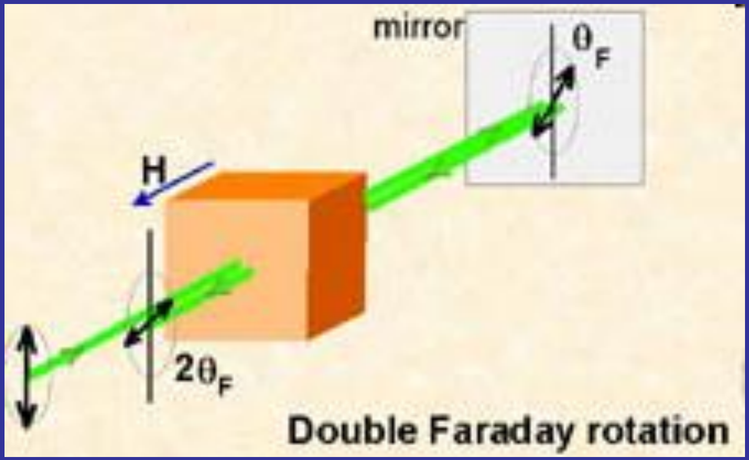
Phase difference $\pi/2$ between x and y components ➔

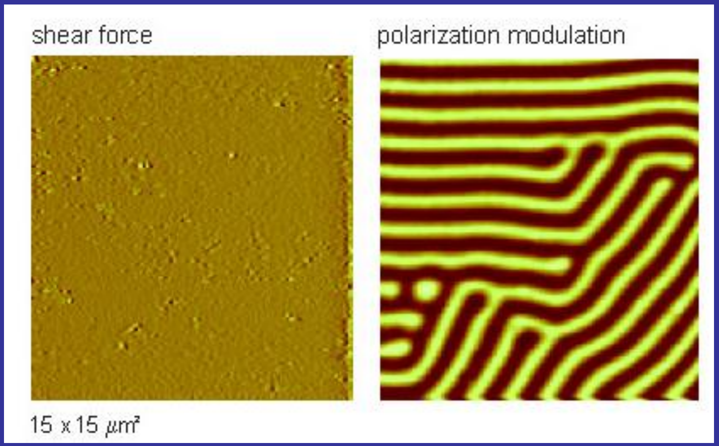
Right and left circularly polarized light !

Time inversion changes direction of B field, as well as left/right

Example: Magneto optic Media







YIG film, Near field magneto-optical imaging
Th. Lacoste, Th. Huser and H. Heinzelmann. Z. Phys. B ***104****, 183 (1997).*

More general

$$D_i(\vec{r}) = \int \varepsilon_{ij}(\vec{r},\vec{r}\,')E_j(\vec{r}\,')d\vec{r}\,'$$

Fourier transform on a lattice:

$$D_i(\vec{k}) = \varepsilon_{ij}(\vec{k},0)E_j(\vec{k}) + \sum_{\vec{h}\neq 0} \varepsilon_{ij}(\vec{k},\vec{h})E_j(\vec{k}-\vec{h})$$

Since k<<h: coupling between D(k) and E(h) small

Furthermore k~0, so taylor expand

Spatial dispersion

$$D_i(\vec{k}) \approx \varepsilon_{ij}(\vec{k})E_j(\vec{k}) \approx \left(\varepsilon_{ij}(0) + i\overleftrightarrow{\gamma}_{ij}\cdot\vec{k}\right)E_j(\vec{k})$$

$$\Rightarrow D_i(\vec{r}) \approx \varepsilon_{ij}E_j(\vec{r}) + \sum_l \gamma_{ijl}\frac{dE_j(\vec{r})}{dr_l} \equiv \tilde{\varepsilon}_{ij}E_j(\vec{r})$$

Symmetry and Response

Dielectric function is a tensor, including spatial dispersion:

$$\hat{\epsilon}_{ij} = \epsilon_{ij} + \sum_k \gamma_{ijk} \frac{\partial}{\partial r_k}$$

For plane waves:

$$\hat{\epsilon}_{ij} = \epsilon_{ij} + i\gamma_{ij},$$

with $\gamma_{ij} = \sum_l \gamma_{ijk} k_l$.

If there is time reversal symmetry

$$\epsilon_{ij} = \epsilon_{ji}$$
$$\gamma_{ijk} = -\gamma_{jik}$$

so that

$$\epsilon = \epsilon_1^s + i\epsilon_2^s$$
$$\gamma = \gamma_1^a + \gamma_2^a$$

If there are no electromagnetic losses ($\int E \cdot \partial D / \partial t dV = 0$):

$$\epsilon_{ij} = \epsilon_{ji}^*$$
$$\gamma_{ijk} = -\gamma_{jik}^*$$

so that

$$\epsilon = \epsilon_1^s + i\epsilon_2^a$$
$$\gamma = \gamma_1^a + \gamma_2^s$$

ϵ_1^s	Birefringence	l	t	i
ϵ_2^s	Linear dichroism		t	i
ϵ_2^a	Faraday effect	l		i
ϵ_1^a	Magnetic circular dichroism			i
γ_1^a	Circular birefringence (opt. act.)	l	t	
γ_2^a	Circular dichroism		t	
γ_2^s	Gyrotropic birefringence	l		
γ_1^s	Gyrotropic dichroism			

l = lossless
t = time reversion invariant
i = space inversion invariant

Relation between the various contributions to the dielectric function, symmetry, and optical phenomena
(l: lossless; t,i: invariant under time reversal, resp. space inversion) :

- Derive the wave equation
- 1.8; 1.12; 1.19

- Derive the response function of a Lorentz oscillator
- 2.3; 2.6;

- 7.1, 7.6, 7.7;