Overview

- Introduction (Fox-Ch1)
  - Response function
  - Optical processes
  - Optical constants
- Waves in solids (Fox-Appendix A)
  - Maxwell equations and wave equation
- Models (Fox-Ch2,3,7)
  - Lorentz model
  - Drude-Lorentz model
  - Transition rates, QM treatment
- Magneto-optical effects, XMCD
- Inelastic light scattering
- Non-linear optics
- Time resolved optics
- Optical modification of matter

Metals

Metals

Lorentz model: Polarization due to bound charges

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = -\frac{e}{m} E(t)$$

Restoring force $\omega_0 = \sqrt{\frac{K}{m}}$

→Leads to single absorption line at the resonance frequency

response function for N identical oscillators:

$$\chi(\omega) = \frac{Ne^2}{\varepsilon_0 m} \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega} = \frac{\omega_p{}^2}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

Metals have ‘free’ electrons: like having no restoring force

Metals – Drude model

Lorentz model without restoring force: Polarization due to free charges

$$\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}=-\frac{e}{m}E(t)$$

Same calculation as before: Calc. position, multiply by charge for P, compare to $P=\varepsilon_0\chi E$

$$x(t)=\frac{e}{m}\frac{E(t)}{\omega^2+i\gamma\omega}\qquad \chi=\frac{NP}{\varepsilon_0 E}=-\frac{Nex}{\varepsilon_0 E}=-\frac{Ne^2}{\varepsilon_0 m}\frac{1}{\omega^2+i\gamma\omega}=-\frac{\omega_p^2}{\omega^2+i\gamma\omega}$$

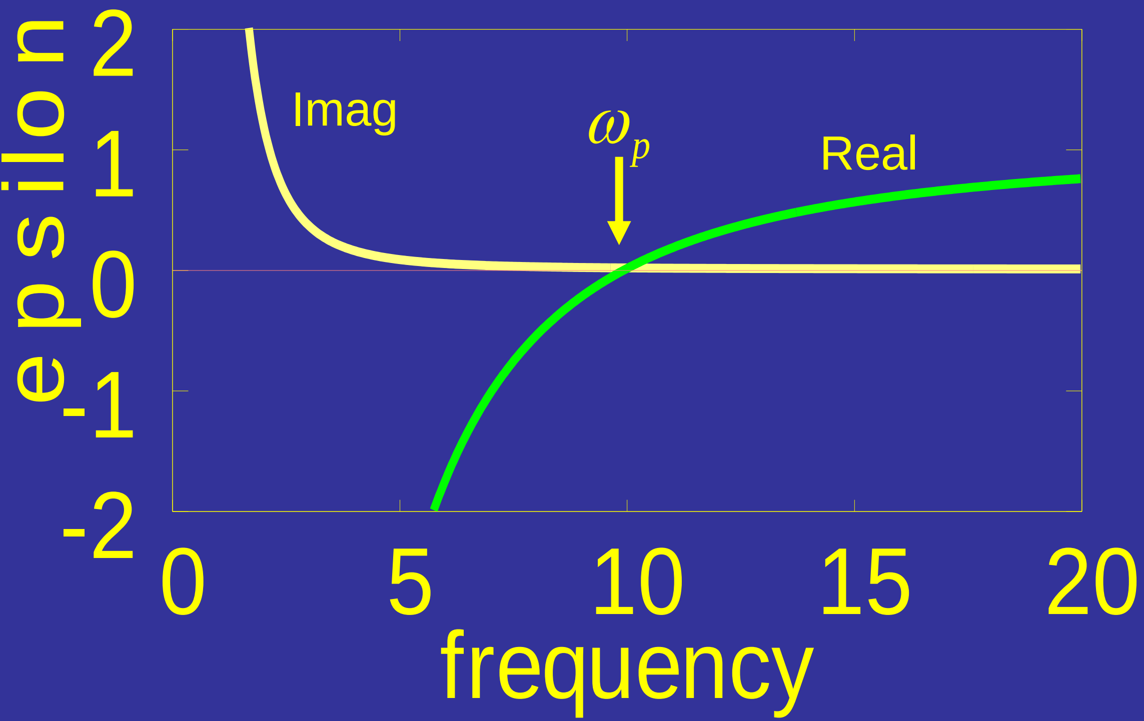
Real and imaginary parts:

$$\omega_p{}^2=\frac{Ne^2}{\varepsilon_0 m}$$

Susceptibility $\chi'=-\frac{\omega_p^2}{\omega^2+\gamma^2}$ $\qquad\chi''=\frac{\omega_p^2\cdot\gamma/\omega}{\omega^2+\gamma^2}$

Dielectric function $\varepsilon'(\omega)=1-\frac{\omega_p^2}{\omega^2+\gamma^2}$ $\qquad\varepsilon''(\omega)=\omega_p^2\frac{\gamma/\omega}{\omega^2+\gamma^2}$

Dielectric Function Metals

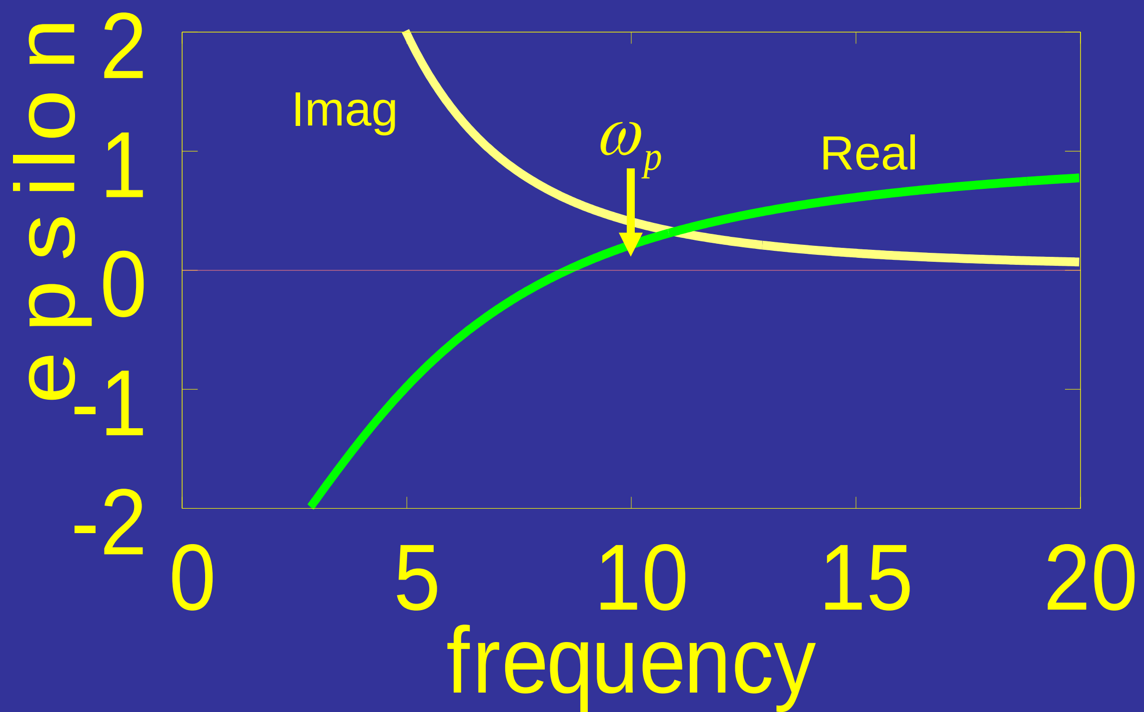


$$\omega_p = 10$$

$$\gamma = 0.1$$

$$\varepsilon'(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + \gamma^2} \qquad \varepsilon''(\omega) = \omega_p^2 \frac{\gamma / \omega}{\omega^2 + \gamma^2}$$

Dielectric Function Metals



$\omega_p = 10$

$\gamma = 5$

$$\varepsilon'(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + \gamma^2} \qquad \varepsilon''(\omega) = \omega_p^2 \frac{\gamma / \omega}{\omega^2 + \gamma^2}$$

Metals

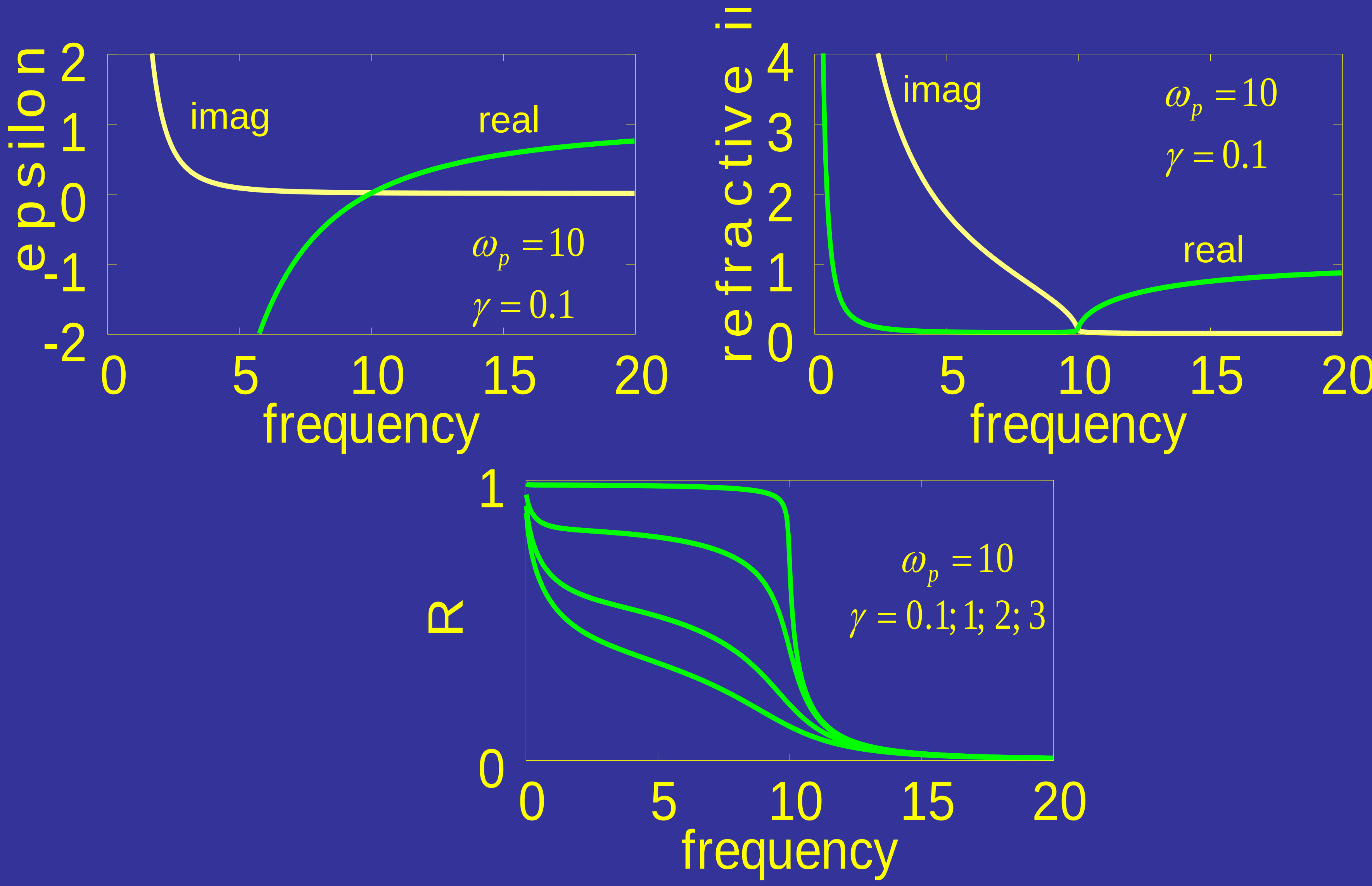
- Damping term due to collisions $\gamma = \frac{1}{\tau}$

$$\varepsilon'(\omega) = 1 - \frac{\omega_p^2 \tau^2}{\omega^2 \tau^2 + 1} \qquad \varepsilon''(\omega) = \omega_p^2 \frac{\tau / \omega}{\omega^2 \tau^2 + 1}$$

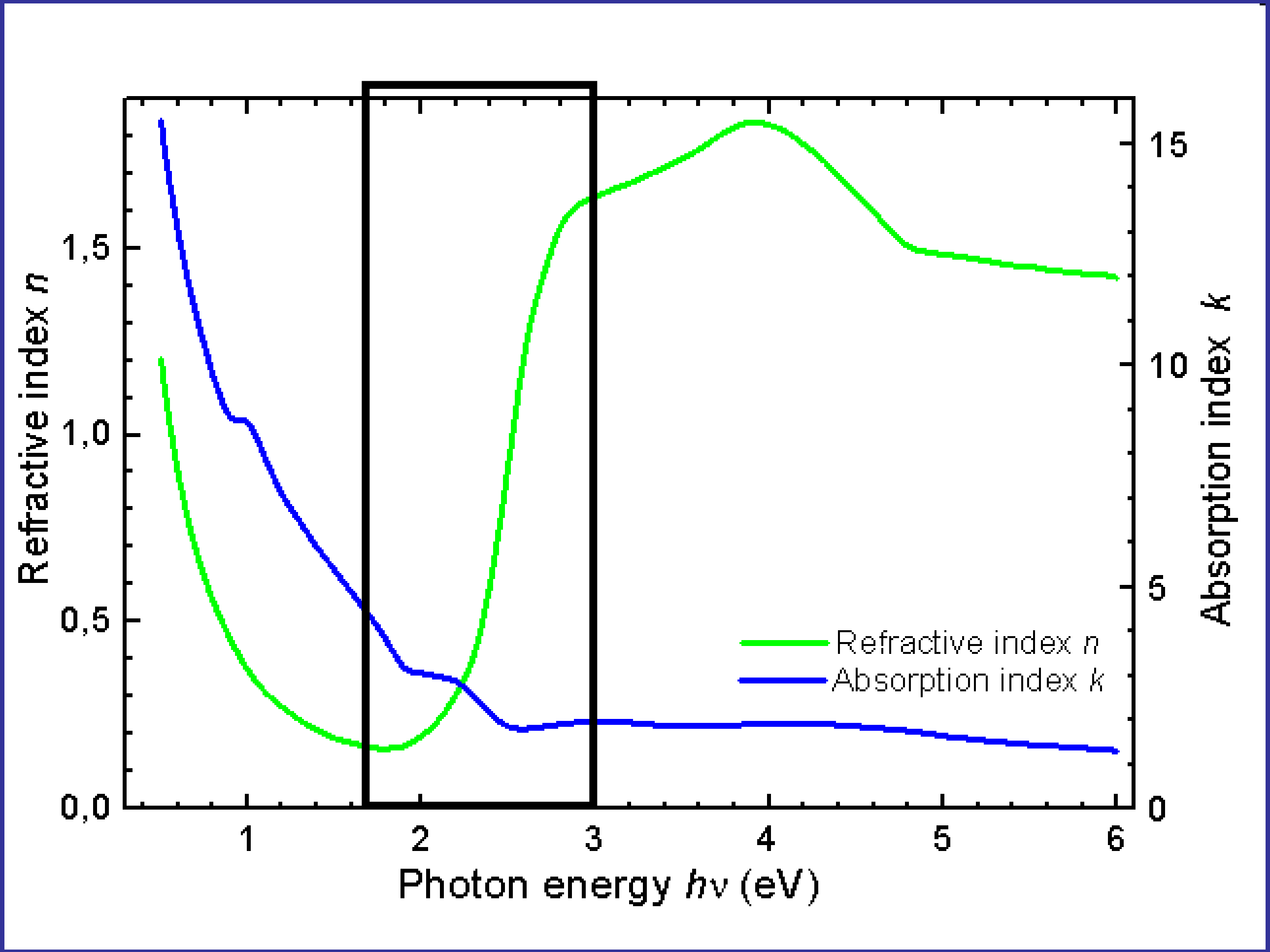
- Scattering rate in metals $\tau \sim 10^{14}\ s^{-1}$
- At optical frequencies $\omega^2 \tau^2 >> 1$
- High frequency limit:

$$\varepsilon'(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \qquad \varepsilon''(\omega) = \frac{\omega_p^2}{\omega^3 \tau}$$

ε,n,R for a metal



Gold

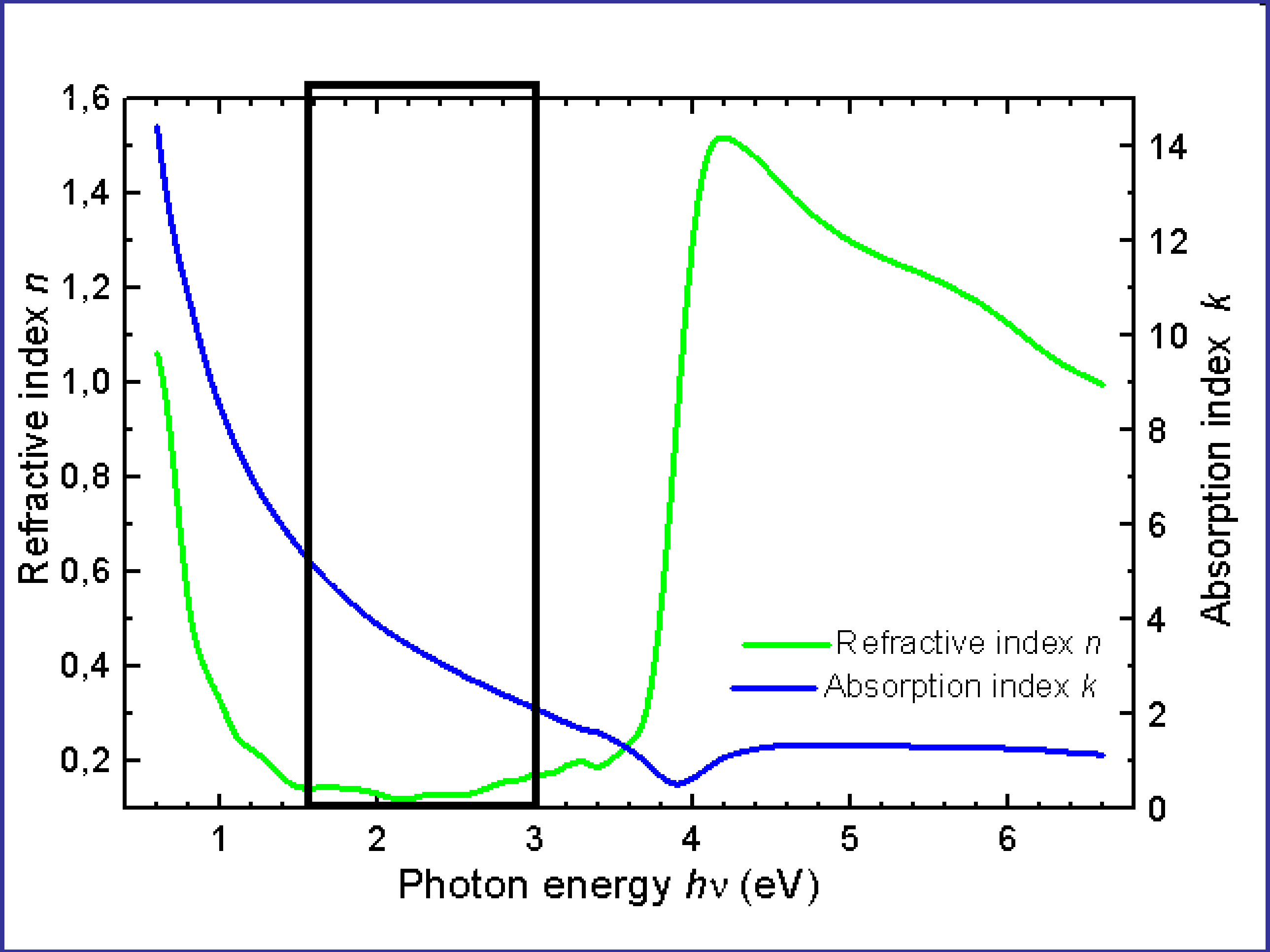


$\omega_p \approx 9\,\mathrm{eV}$

$5d^{10}6s^1$

http://www.ioffe.rssi.ru/SVA/NSM

Silver



$\omega_p \approx 9\,\mathrm{eV}$

$4d^{10}5s^{1}$

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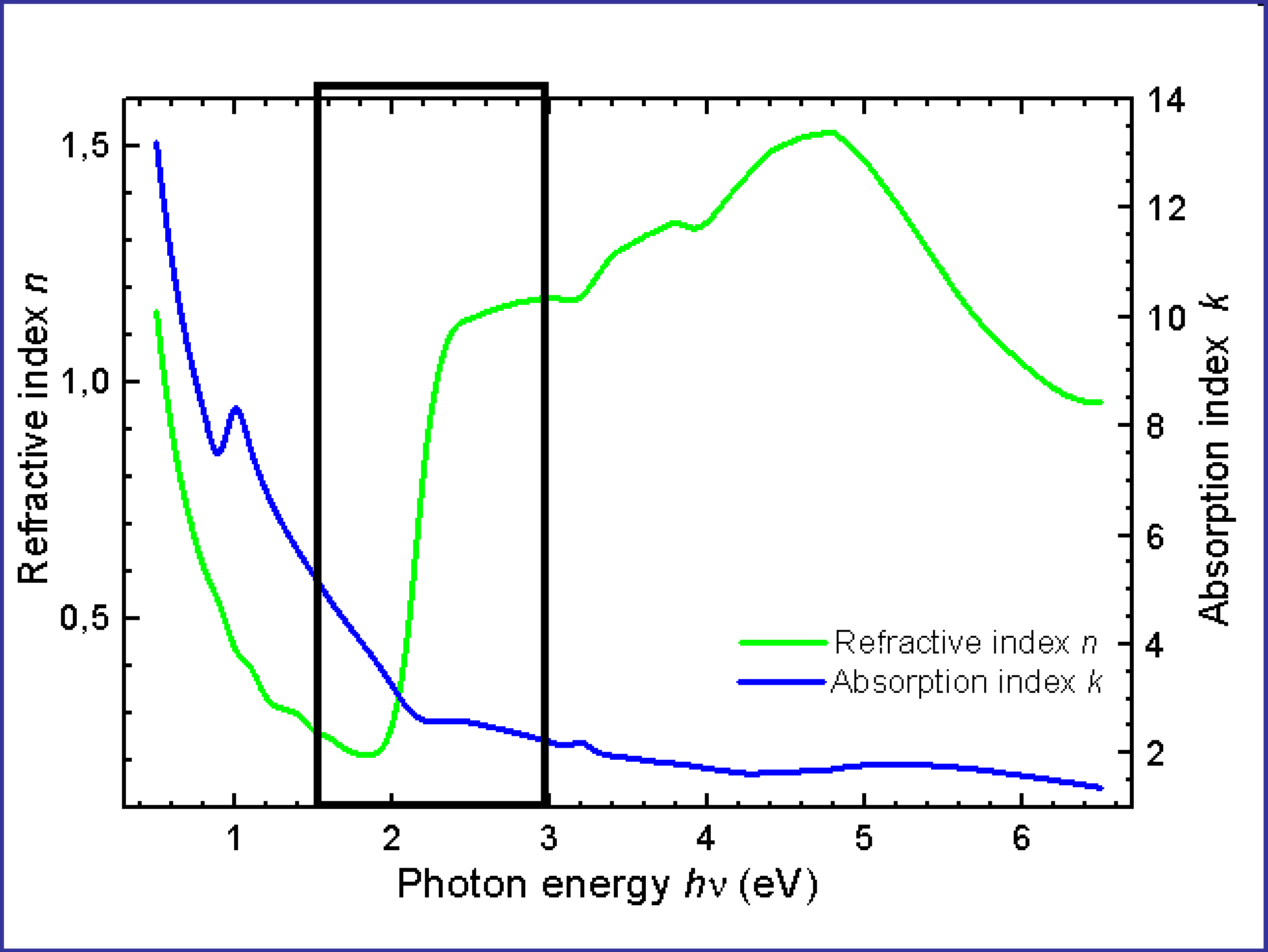
Copper

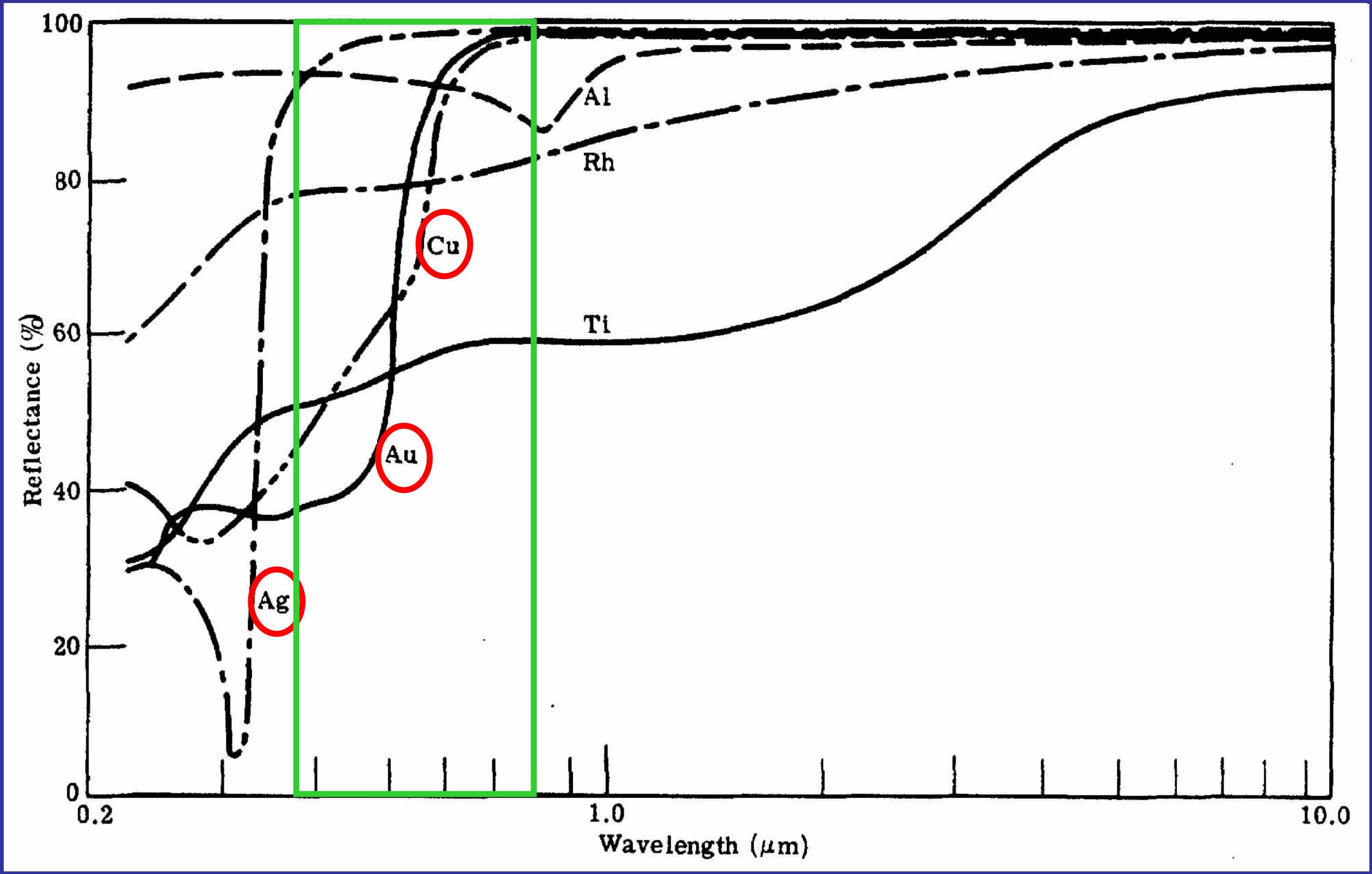


$\omega_p \approx 11\,\text{eV}$

$3d^{10}4s^1$

http://www.ioffe.rssi.ru/SVA/NSM

Reflectance



Typical values

- Metals:
  N ~ 10^{21} cm^{-3} → $\omega_p/2\pi$ ~ 10^{15} Hz → λ = 300 nm (in the UV)
- Doped semiconductors:
  N ~ 10^{17} cm^{-3} → $\omega_p/2\pi$ ~ 10^{13} Hz → λ = 30 μm (in the FIR)

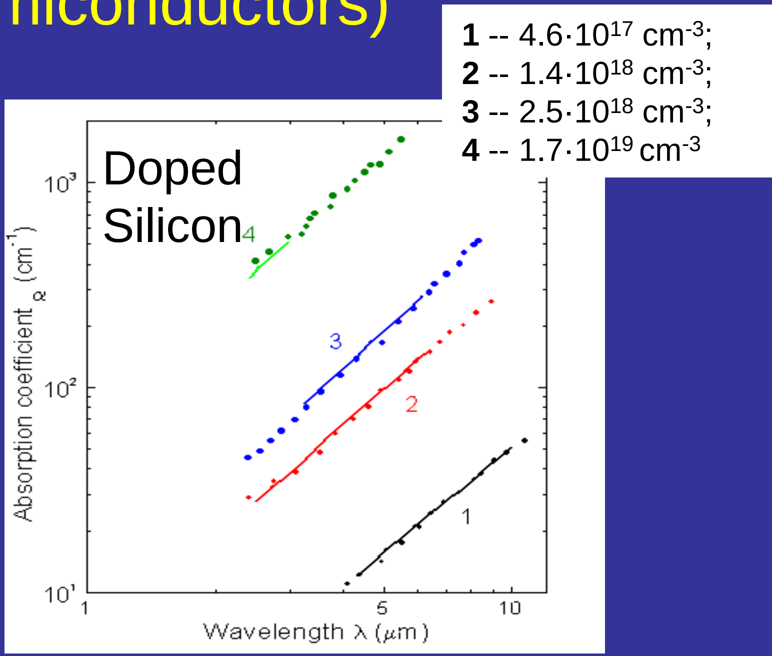
absorption coefficient

High frequencies (infrared for doped semiconductors)

$$\varepsilon'(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \approx 1 \qquad \varepsilon''(\omega) = \frac{\omega_p^2}{\omega^3 \tau}$$

$$n''(\omega) \approx \sqrt{\tfrac{1}{2}\left(\sqrt{1+\varepsilon_r''^2} - 1\right)} \approx \frac{1}{2}\varepsilon_r''$$

$$\alpha(\omega) \approx \frac{1}{2}\frac{\omega_p^2}{\omega^2 \tau} \sim \lambda^2$$



$$n'(\omega) = \sqrt{\tfrac{1}{2}\left(\sqrt{\varepsilon_r'^2 + \varepsilon_r''^2} + \varepsilon_r'\right)} \quad n''(\omega) = \sqrt{\tfrac{1}{2}\left(\sqrt{\varepsilon_r'^2 + \varepsilon_r''^2} - \varepsilon_r'\right)}$$

Drude conductivity

Previously $$\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}=-\frac{e}{m}E$$

Expressed in velocity $$\frac{dv}{dt}+\gamma v=-\frac{e}{m}E$$

velocity $$v=-\frac{e\tau}{m}\frac{1}{1-i\omega\tau}E$$

Current $$J=-Nev=\frac{Ne^2\tau}{m}\frac{1}{1-i\omega\tau}E\equiv\sigma E$$

Conductivity $$\sigma(\omega)=\frac{Ne^2\tau}{m}\frac{1}{1-i\omega\tau}=\frac{\sigma_0}{1-i\omega\tau}=-i\varepsilon_0\omega\left(\varepsilon_r(\omega)-1\right)$$

DC Conductivity $$\sigma(0)=\sigma_0=\frac{Ne^2\tau}{m}$$

- Derive the wave equation
- 1.8; 1.12; 1.19

- Derive the response function of a Lorentz oscillator
- 2.3; 2.6;

- 7.1, 7.6, 7.7;