Condensed Matter Physics 2 - Problem Set 3

In this problem, we will look at the phase transitions of Ising ferro- and anti-ferromagnets. To begin, consider the free energy functional of the form

$$F = -\frac{1}{2}a\tau m^2 + bm^4.$$

Here, m is the magnetization and the order parameter of interest and $\tau = T_c - T$.

1. Plot the free energy for different values of a, b, and τ to see what happens above, at, and below the phase transition.
2. Minimize the free energy and solve for the magnetization below and above the critical temperature. Plot the magnetization as well as the associated free energy functional for different τ.
3. Derive an expression for the entropy of this system and plot it as a function of temperature.
4. Using the entropy, find the heat capacity of the system. What happens at the phase transition?
5. By adding a $-hm$ term to the free energy form, use the above approach to obtain an expression for the susceptibility of the system above and below the critical temperature.

For an Ising antiferromagnet, the ground state is two spin sublattices with opposing spins. Instead of using the magnetization for the order parameter, we use the staggered magnetization $l = m_1 - m_2$.

1. Using the fact that symmetry requires that the Hamiltonian be invariant under interchange of the sublattices, write down the free energy functional for an Ising antiferromagnet in the absence of an external field.
2. Using the same invariance described above, what are the additional terms that result from an external field coupling to the staggered magnetization.
3. Again, minimize the free energy function to determine the staggered magnetization and associated free energy.
4. Find the susceptibility and compare it to the one you obtained for the ferromagnet.
5. Generalize the above approach to write down the most general free energy functional allowed by symmetry in the presence of an external field. Include terms associated with the average magnetization $m = m_1 + m_2$. The free energy functional should now be a function of l and m. Minimize this function against m and find the associated free energy and susceptibility as a function of l. Compare this to the result of part 5 for the ferromagnet case.