Condensed Matter Physics 2 - Problem Set 3

In this problem, we will look at how screening changes the nature of quasiparticle excitations in systems, beginning with the former and the Thomas-Fermi approximation. The Thomas-Fermi approximation can be thought of in the following way. Suppose a metal has a Fermi energy at E_F. Now consider an isolated piece of the same material with the band structure shifted due to some external potential U. When these two samples are contacted, the overall Fermi energy is equalized, and the additional electron density is n_{ind}. This new Fermi level is content through the entire volume.

So, suppose that the electrons in the metal are subjected to a potential of the form

$$U(\mathbf{r}) = A(\mathbf{q}) \exp(i\mathbf{q} \cdot \mathbf{r}) + cc.$$

1. What is the electric field associated with the potential U?

2. The resulting induced charge density is given by $n_{ind} = -n(E_F) U(\mathbf{r})$ with $n(E_F)$ being the charge density at the original E_F. The indicted polarization and density are related by $\nabla \cdot \mathbf{P}_{ind} = en_{ind}$. Show that $\mathbf{P}_{ind}$ is given by

$$\mathbf{P}_{ind}(\mathbf{r}) = i\frac{en(E_F)}{q^2}\mathbf{q}A(\mathbf{q}) \exp(i\mathbf{q} \cdot \mathbf{r}) + cc.$$

3. Use the results from parts 1 and 2 to show that the static dielectric function is given by

$$\epsilon(\mathbf{q}) = 1 + \frac{4\pi e^2 n(E_F)}{q^2}.$$

   We can set $k_T = \sqrt{4\pi e^2 n(E_F)}$ and write $\epsilon(\mathbf{q}) = 1 + k_T^2/q^2$. Here, k_T is known as the Thomas-Fermi screening wave vector.

4. What does k_T physically represent? To find out, consider a medium with a point charge (Ze) placed at the origin. The total potential energy of this system can be written (by employing a Fourier transform) as

$$U(\mathbf{r}) = -\frac{1}{(2\pi)^3} \int \frac{1}{\epsilon(q)} \frac{4\pi Z e^2}{q^2} \exp(i\mathbf{q} \cdot \mathbf{r}) \, d\mathbf{q}.$$

   Use the result of part 3 and evaluate the integral to find the form of $U(\mathbf{r})$. What is the physical interpretation of k_T?

5. Now lets see how screening can impact the quasiparticle excitations of a system. We will do this using a different formulation of the (dynamic) dielectric function, the *Lindhard dielectric function* given by

$$\epsilon(\mathbf{q}, \omega) = 1 + \frac{8\pi e^2}{Vq^2} \sum_{\mathbf{k}} \frac{f(\mathbf{k}) - f(\mathbf{k}+\mathbf{q})}{E(\mathbf{k}+\mathbf{q}) - E(\mathbf{k}) - \hbar\omega - i\eta}.$$

   To begin, consider a simple semiconductor characterized by a parabolic conduction band. In this case, The Lindhard function can be solved analytically, and for small q

$$\epsilon(\mathbf{q}, \omega) = 1 - \frac{\omega_p^2}{(\omega + i\eta/\hbar)^2} - \frac{3}{5} \frac{\omega_p^2}{(\omega + i\eta/\hbar)^4} \frac{\hbar^2 k_F^2}{m} q^2 - \cdots$$

   Choosing appropriate values of ω_p, η, k_F, and m, plot ϵas a function q and ω. The roots of the dielectric function are related to the plasmon excitations of the system. Isolate values of q and ω for which $\epsilon = 0$. How does ω vary as a function of q?

6. Using part 3, modify the expression in part 5 to include a Thomas-Fermi screening term. Again, plot ϵas a function q and ω. Isolate values of q and ω for which $\epsilon = 0$. How does ω vary as a function of q?

The dramatic difference in q-dependence between two systems shows that strong screening leads to a new type of quasiparticle excitation.