

Magnetism

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Optical Condensed Matter Physics

- Crystal fields
- d,f electrons
- angular momentum quenching
- p-d interactions, splitting of levels
- Spin peierls
- Jahn Teller
- Charge, orbital, spin ordering

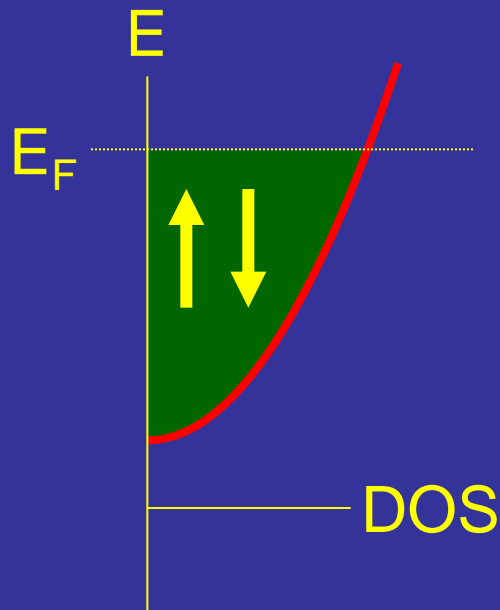
Today

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Magnetism in metals

Free electron gas

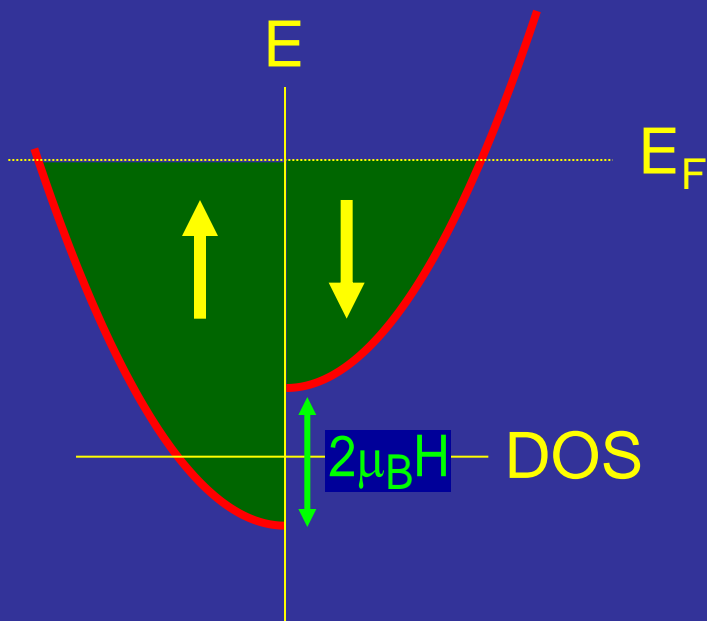
No field: $E = \frac{\eta^2 k^2}{2m^*}$ $E_F = \frac{\eta^2}{2m^*} (3\pi^2 n)^{2/3}$ $D(E) = \frac{1}{2\pi^2} \left(\frac{2m^*}{\eta^2} \right)^{3/2} \sqrt{E}$



- Pauli paramagnetism, Landau diamagnetism
- Spontaneous spin polarization (Stoner)
- RKKY

Pauli paramagnetism

H ≠ 0 :
$$E = \frac{\eta^2 k^2}{2m^*} \pm \frac{1}{2} g_0 \mu_B H$$



$$N_{\uparrow} = \frac{1}{2} \int_{-\mu_B H}^{E_F} D(E + \mu_B H) dE$$

$$\approx \frac{1}{2} \left(\int_0^{E_F} D(E) dE + \mu_B H \cdot D(E_F) \right)$$

$$N_{\downarrow} \approx \frac{1}{2} \left(\int_0^{E_F} D(E) dE - \mu_B H \cdot D(E_F) \right)$$

Pauli:
$$M = \mu_B (N_{\uparrow} - N_{\downarrow})$$

$$= \mu_B^2 D(E_F) H = \frac{3n\mu_B^2}{2kT_F} H$$

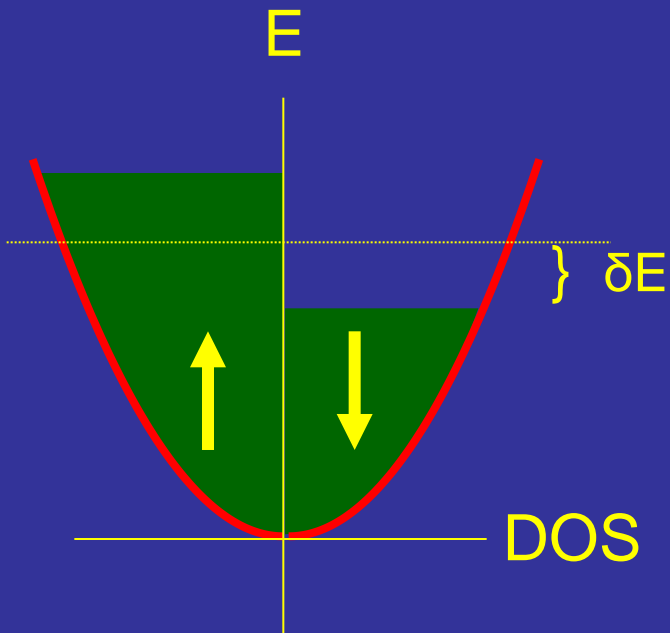
Landau (dia):
$$M = -\frac{n\mu_B^2}{2kT_F} H$$



$$\chi_e = \frac{n\mu_B^2}{kT_F}$$

Stoner magnetism

Spontaneous spin polarization



If $[1 - UD(E_F)] < 0$ then $\Delta E_{\text{tot}} < 0 \Rightarrow$ Magnetic ground state

Happens for strong Coulomb and high D.O.S.

If spin split then 'internal' field $H = \lambda M$

$$\text{Cost in kinetic energy : } \Delta E_k = \left[\frac{1}{2} D(E_F) \cdot \delta E \right] \cdot \delta E$$

Magnetization :

$$n_{\uparrow} = \frac{1}{2} (n + D(E_F) \cdot \delta E); \quad n_{\downarrow} = \frac{1}{2} (n - D(E_F) \cdot \delta E)$$

$$M = \mu_B (n_{\uparrow} - n_{\downarrow}) = \mu_B D(E_F) \cdot \delta E$$

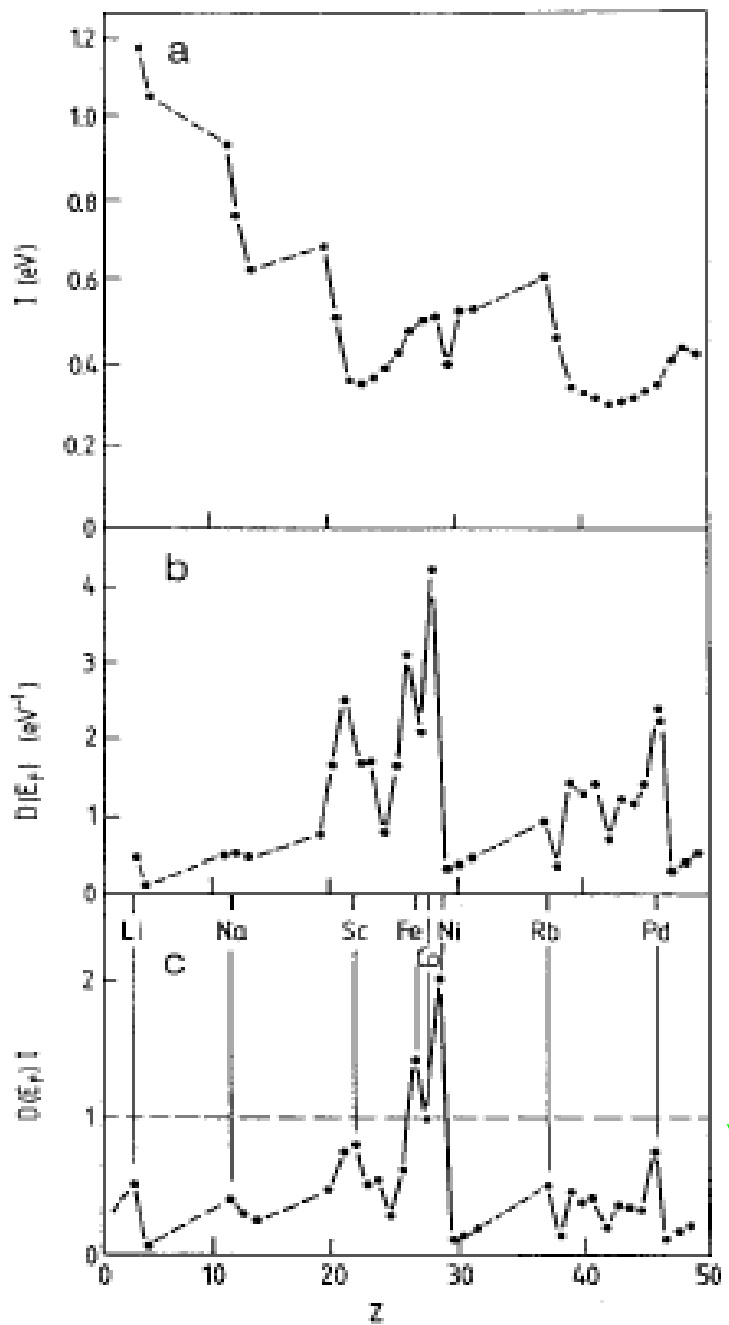
Field energy

$$\begin{aligned} \Delta E_p &= - \int_0^M B dM' = - \int_0^M \mu_0 (\lambda M') dM' = - \frac{1}{2} \mu_0 \lambda M^2 \\ &= - \frac{1}{2} \mu_0 \mu_B^2 \lambda (n_{\uparrow} - n_{\downarrow})^2 = - \frac{1}{2} U (n_{\uparrow} - n_{\downarrow})^2 = - \frac{1}{2} U [D(E_F) \cdot \delta E]^2 \end{aligned}$$

Total energy $\Delta E_{\text{tot}} = \Delta E_k + \Delta E_p$

$$\Delta E_{\text{tot}} = \frac{1}{2} D(E_F) \cdot \delta E^2 [1 - UD(E_F)]$$

Stoner criterium



Exchange interaction

Density of States at E_F

Product

In agreement with
Fe, Ni, Co ferromagnets

Stoner criterium

Stoner magnetism

- If $UD(E_F) < 1$ then still change in susceptibility

Total energy in external field B : $\Delta E_{\text{tot}} = \Delta E_{\text{k}} + \Delta E_{\text{p}} - MB$

$$M = \mu_B (n_{\uparrow} - n_{\downarrow}) = \mu_B D(E_F) \cdot \delta E$$

$$\Delta E_{\text{tot}} = \frac{1}{2} \frac{M^2}{\mu_B^2 D(E_F)} [1 - UD(E_F)] - M \cdot B$$

Minimization w.r.t. M leads to

$$M = \frac{\mu_B^2 D(E_F)}{[1 - UD(E_F)]} B$$

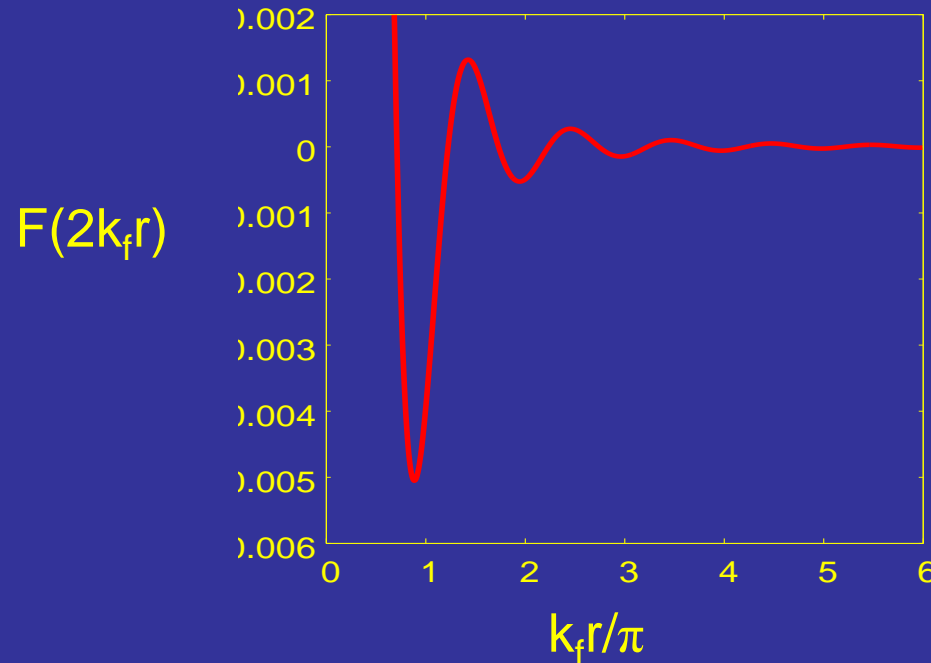
Susceptibility $\chi = M/H = M/(B/\mu_0)$

$$\chi = \frac{\mu_0 \mu_B^2 D(E_F)}{[1 - UD(E_F)]} = \frac{\chi_{\text{Pauli}}}{[1 - UD(E_F)]}$$

\Rightarrow Enhanced susceptibility

Spatially varying fields

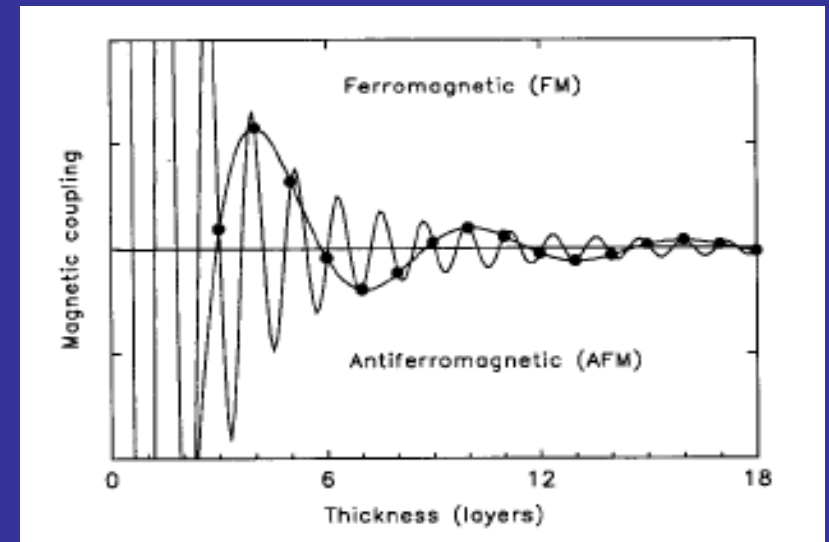
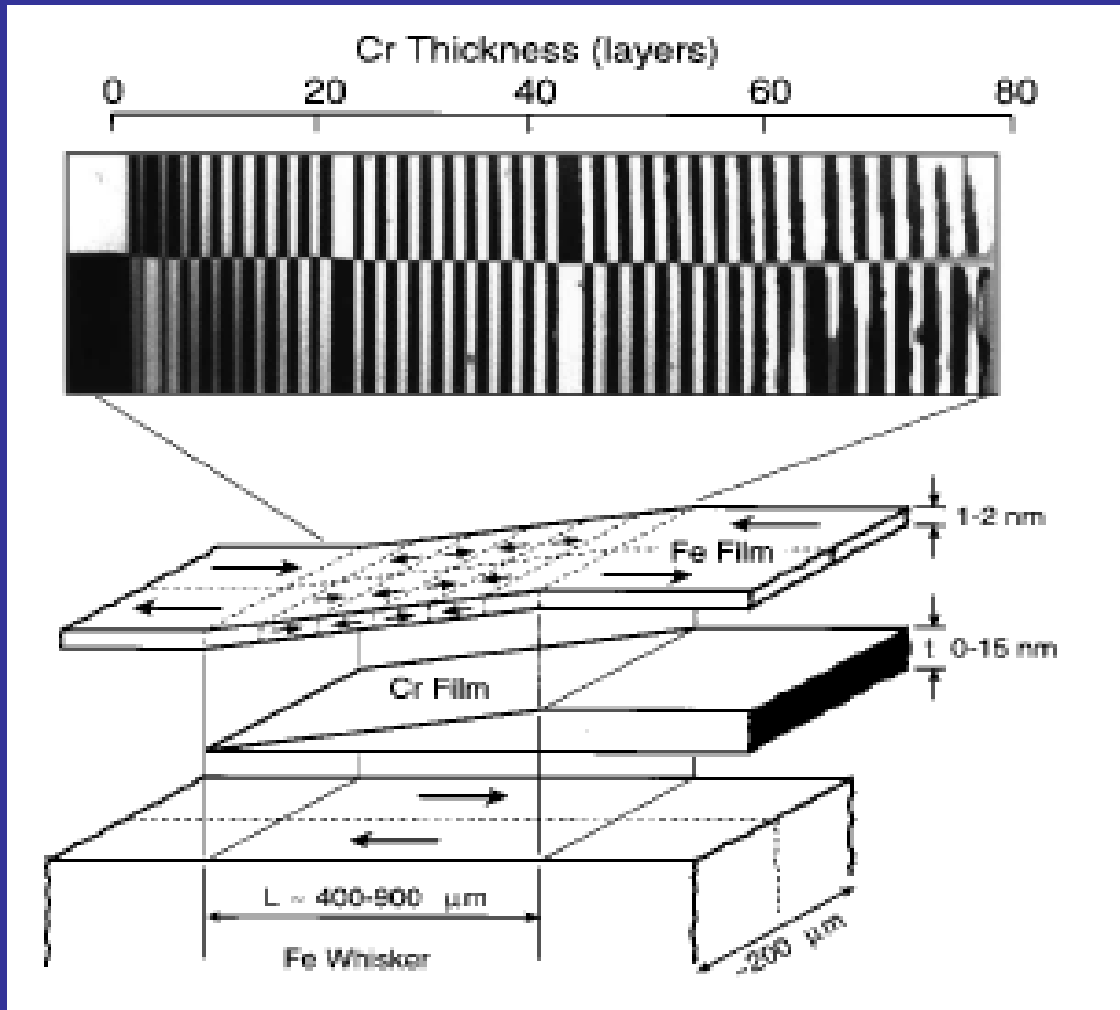
- RKKY interaction (*Ruderman-Kittel-Kasuya-Yosida*) (par. 7.7)



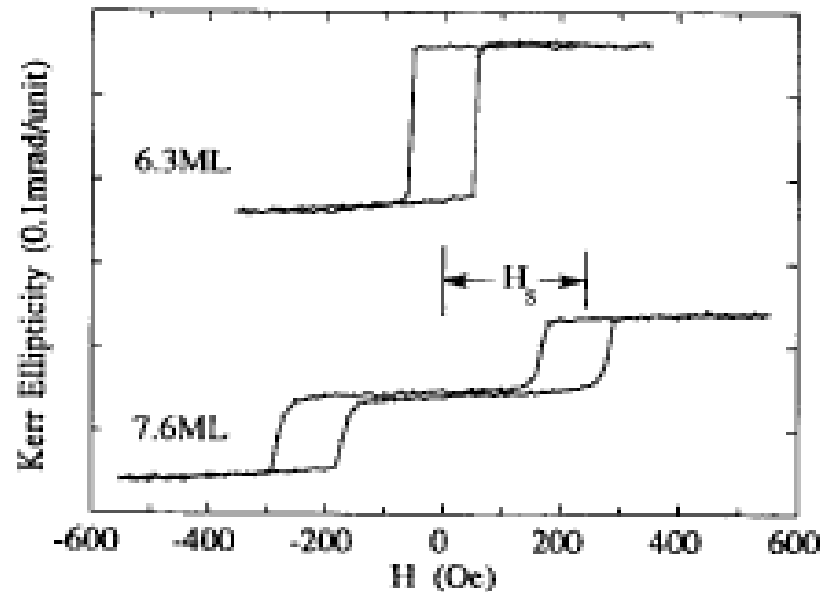
$$H(r) = H \delta(r)$$

$$\chi(r) = \frac{2}{\pi} k_f^3 \chi_{\text{pauli}} F(2k_f r) \stackrel{x \gg 1}{=} -\frac{2}{\pi} k_f^3 \chi_{\text{pauli}} \frac{\cos(2k_f r)}{(2k_f r)^3}$$

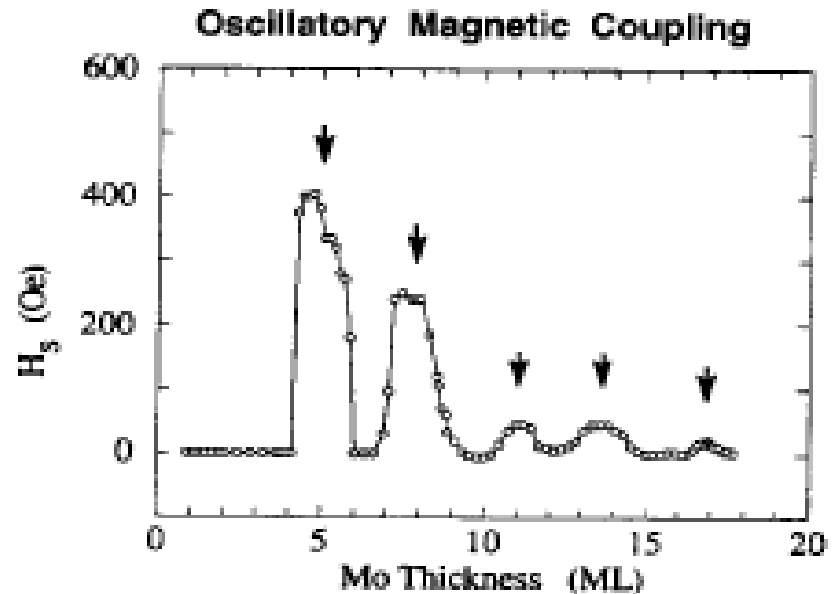
RKKY interaction



RKKY interaction



(a)



(b)

Figure 38. Magnetic oscillations at Fe/Mo/Fe(100) trilayers determined by the SMOKE (Qiu *et al.* 1992b). (a) Hysteresis loops characteristic of parallel and antiparallel coupling (top and bottom). H_s is the magnetic field required to force antiparallel layers parallel. Adding just slightly more than a monolayer to the Mo spacer reverses the magnetic orientation. (b) Alternating antiparallel and parallel coupling (arrows and baseline respectively).

Ch. 5 Ordered magnetism

- Ferromagnets
 - Antiferromagnets
 - Ferrimagnets
 - Helical order
-
- Weiss model (Mean field, ferromagnetic, $L=0$)

$$H = -\sum_{ij} J_{ij} S_i \cdot S_j + g\mu_B \sum_i S_i \cdot H$$

Mean field model

- Each spin feels the field of all other

$$\text{Field of other spins: } H_{mf} = \lambda M$$

$$E_i \stackrel{m.f.}{=} -2 \sum_j J_{ij} S_i \cdot \langle S_j \rangle \stackrel{n.n.}{=} -2Jz \langle S \rangle \cdot S_i == m_i H_{mf} = -g\mu_B S_i \cdot H_{mf}$$

$$\left. \begin{aligned} H_{mf} &= \frac{2Jz}{g\mu_B} \langle S \rangle == \lambda M \\ \langle S \rangle &= \frac{M}{ng\mu_B} \end{aligned} \right\} \lambda = \frac{2Jz}{(g\mu_B)^2 n}$$

Above ordering temperature

Paramagnetic $\rightarrow M = \chi_{\text{curie}} H_{\text{total}} = \frac{C}{T} H_{\text{total}}$

$$C = \frac{n(p\mu_B)^2}{3k}$$

$$p = g_J \sqrt{J(J+1)}$$

Field of other spins : $H_{\text{mf}} = \lambda M$

Total field on a spin : $H_{\text{total}} = H_{\text{ext}} + H_{\text{mf}}$

Magnetization : $M = \chi_{\text{curie}} H_{\text{total}} = \frac{C}{T} (H_{\text{ext}} + H_{\text{mf}}) = \frac{C}{T} (H_{\text{ext}} + \lambda M)$

$$\frac{M}{H_{\text{ext}}} \equiv \chi = \frac{C/T}{1 - \lambda \frac{C}{T}} = \frac{C}{T - \lambda C} = \frac{C}{T - T_C}$$

T_C : Curie - Weiss temperature

$$\left. \begin{array}{l} T_C = \lambda C \\ C = \frac{n(p\mu_B)^2}{3k} \\ \lambda = \frac{2Jz}{(g\mu_B)^2 n} \end{array} \right\} \begin{array}{l} T_c = \frac{Jzp^2}{6k} \\ J = \frac{6}{zp^2} kT_c \end{array}$$

Below T_C : Spontaneous order

Spontaneous order $\rightarrow H_{ext} = 0; M \neq 0; H_{mf} = \lambda M$

Field of other spins : $H_{mf} = \lambda M$

Total field on a spin : $H_{total} = H_{mf}$

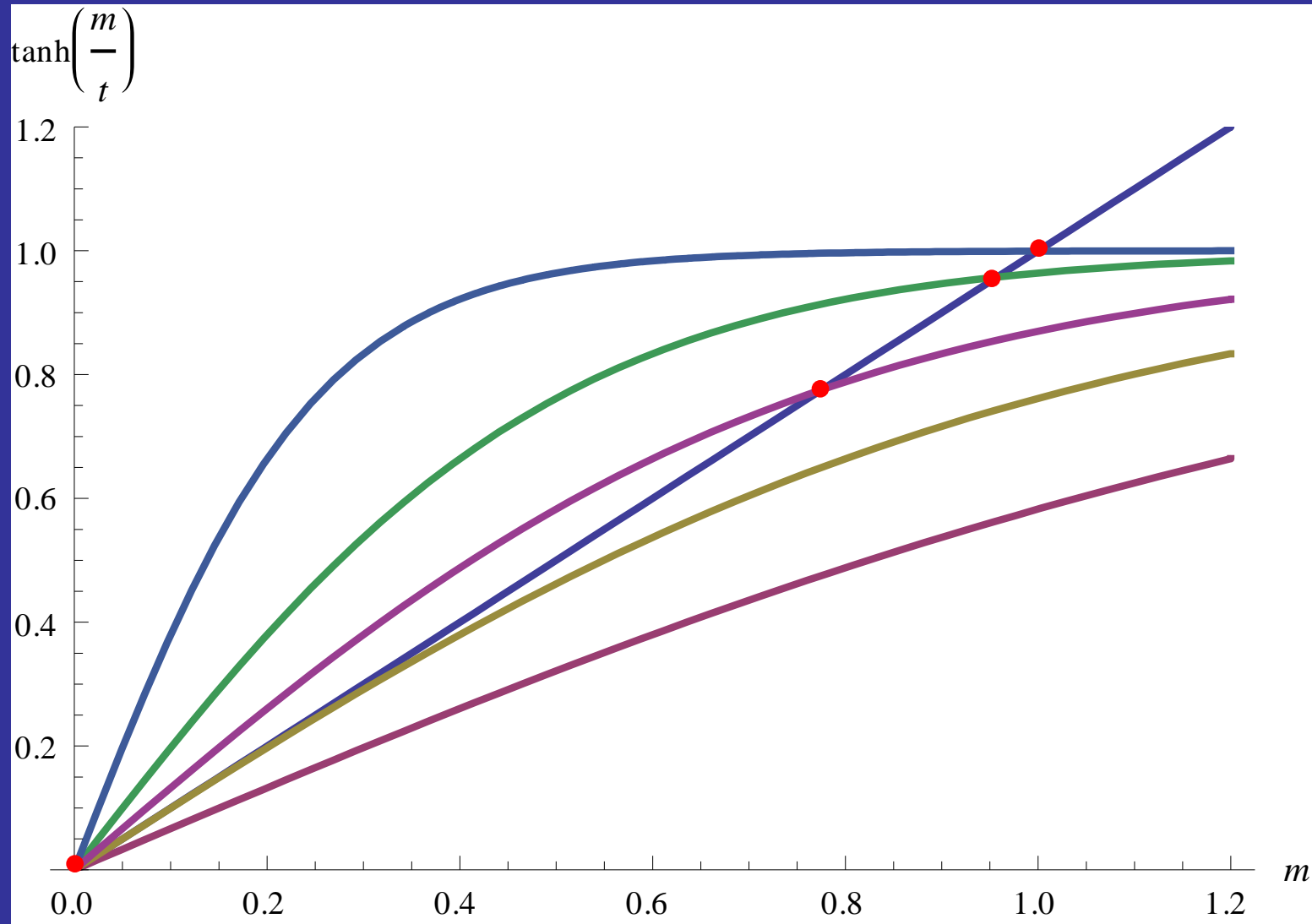
Magnetization : $M = n g_J \mu_B J B_J(x)$ with $x = \frac{g_J \mu_B J H_{mf}}{kT} = \frac{g_J \mu_B J \lambda M}{kT}$

For $g = 2; J = 1/2$: $M = n \mu_B \tanh\left(\frac{\mu_B \lambda M}{kT}\right)$

or $m = \tanh(m/t)$

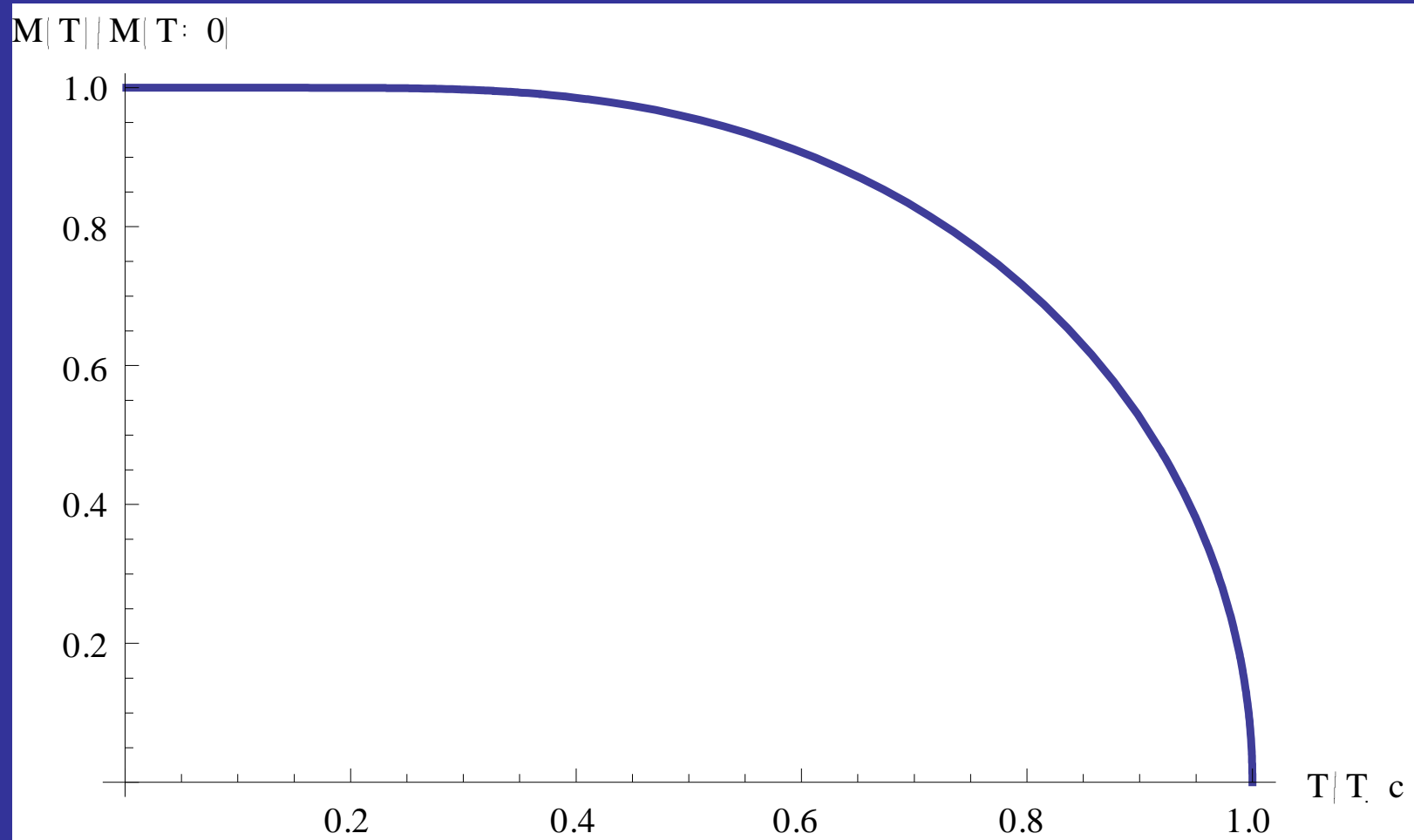
with $m = \frac{M}{n \mu_B}$ and $t = \frac{kT}{\lambda n \mu_B^2}$

Below T_C : Spontaneous order



t
0.25
0.5
0.75
1
1.5

Below T_C : Spontaneous order



Numerically solved $m = \tanh(m/t)$

Ferromagnets

	z	n [10^{22} cm $^{-3}$]	g	p	C [K]	T _c [K]	J [meV]	λ
Fe	8	8.5	2	5.4	0.51	1043	2.3	2045
Co	12	9	2	4.8	0.43	1388	2.3	3228
Ni	12	9.1	2	3.2	0.19	627	0.6	3300
Gd	12	3	2	8.0	0.40	293	0.2	733

	M(0) [gauss]	M(0)/N μ_B	H _{mf} [10^6 gauss]
Fe	1740	2.22	3.6
Co	1446	1.72	4.7
Ni	510	0.606	1.7
Gd	2060	7.63	1.5