1D systems.

1] $H = \sum_{\langle ij \rangle} J \vec{S}_i \cdot \vec{S}_j$

'semi-classical' assume G.S. Néel. then excitation energy $E_N = JS^2 \frac{\pi^2}{N}$

(see bloch wall discussion L5).

so $E_N \to 0$ for $N \to \infty$.

No energy cost to make $\Delta S = 1$ excitation

one spin flipped

2] 1D crystal.

displacement one phonon mode ~~annih~~ ~~create~~ create annil

$$u = \sqrt{\frac{\hbar}{2M\omega}} \cdot (b^+ + b).$$

then average displ. $\langle u^2 \rangle = \frac{\hbar}{2M\omega} \langle (b^+ + b)^2 \rangle =$

$$\frac{\hbar}{2M\omega} \langle (b^+ b^+ + bb + b^+ b + bb^+) \rangle =$$

$$\frac{\hbar}{2M\omega} (2 \langle b^+ b \rangle + 1) = \frac{\hbar}{M\omega}(n + \tfrac{1}{2})$$

since $[b, b^+] = 1$

$bb^+ = b^+ b + 1$

in 1D x-tal @ $T=0$ (i.e. $n=0$).

$N(k) = \frac{1}{2\pi}$

$$\langle u^2 \rangle = \sum_k \frac{\hbar}{M\omega_k} \left(\langle b_k^+ b_k \rangle + \frac{1}{2} \right) = \int \frac{dk}{2\pi} \cdot \frac{\hbar}{M\omega_k} \cdot \frac{1}{2}.$$

for small k: $\omega_k = v_{sound} k \Rightarrow$

$$\langle u^2 \rangle = \int_0 \frac{dk}{2\pi} \cdot \frac{1}{2} \frac{\hbar}{M v_s k} \Rightarrow \text{logarithmically divergent!!}$$

i.e. @ $T=0$ $\langle u^2 \rangle \to \infty$

at finite T even worse (include $n = \frac{1}{e^{\hbar\omega_k/kT}-1}$).

in 2D at $T=0$ no divergence $\Rightarrow$ stable
at any $T \neq 0$ divergent $\Rightarrow$ unstable.

Mermin Wagner theorem:
In one- and two-dimensional systems
continuous symmetries can not be broken spontaneously.
(1D, 2D: $T=0$); (in 1D also not at finite T).

3) excitations in 1D chain; fractionalization of degrees of freedom.

$\uparrow\ \downarrow\ \uparrow\ \downarrow\ \uparrow\ \downarrow\ \uparrow\ \downarrow\ \uparrow$ (N)

$\uparrow\ \downarrow\ \uparrow\ \circ\ \uparrow\ \downarrow\ \uparrow\ \downarrow\ \uparrow$ (N−1) charge + spin gone
$\Delta q = 1 \quad \Delta S = 1/2$.

$\uparrow\ \downarrow\ \uparrow\ \uparrow\ \circ\ \downarrow\ \uparrow\ \downarrow\ \uparrow$ one hop

$\uparrow\ \downarrow\ \uparrow\ \uparrow\ \downarrow\ \circ\ \uparrow\ \downarrow\ \uparrow$ 2 hops

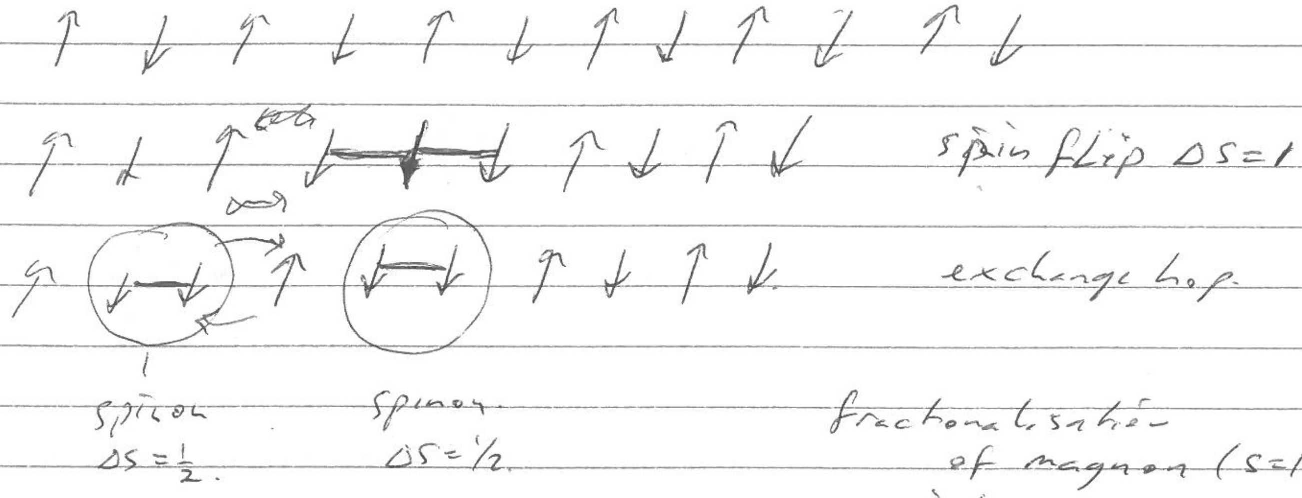
$\uparrow\ \downarrow\ \uparrow\ \uparrow\ \downarrow\ \uparrow\ \circ\ \downarrow\ \uparrow$ 3 hops, etc.

spin excitation
effectively $\Delta S = \frac{1}{2}$.

<u>spinon</u>.

charge.
<u>holon</u>

4) Rather drastic taking out particle ~~a~~ $H = \sum_i J S_i S_{i+1}$
spinons are natural to 1D systems.



fractionalisation
of magnon $(S=1)$
into spinons.

Magnons in ~~(3D)~~ 2D system La_2CuO_4. (ordered!)

$\boxed{\text{sheet 1}}$ La_2CuO_4 well defined magnon branches.
theory like last time (spin waves).

2-spinon excitations ~~Magnons~~ in 1D system $KCuF_2$ (spin-liquid).

(cmp. supercond.; spectroscopically one can not
excite single quasi-particle
only break cooper-pair → 2 quasi-particles
⇒ 2Δ.)

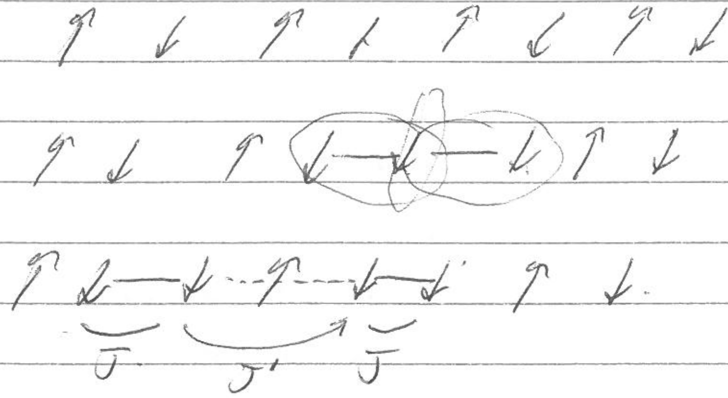
$\boxed{\text{sheet 2, 3}}$ $KCuF_3$.

5) Bound states suppose one has ~~anti~~ ferromagnetic
next nearest neighbor exchange

$$H = \sum_i J\, S_i S_{i+1} - \sum_i J' S_i S_{i+2}.$$

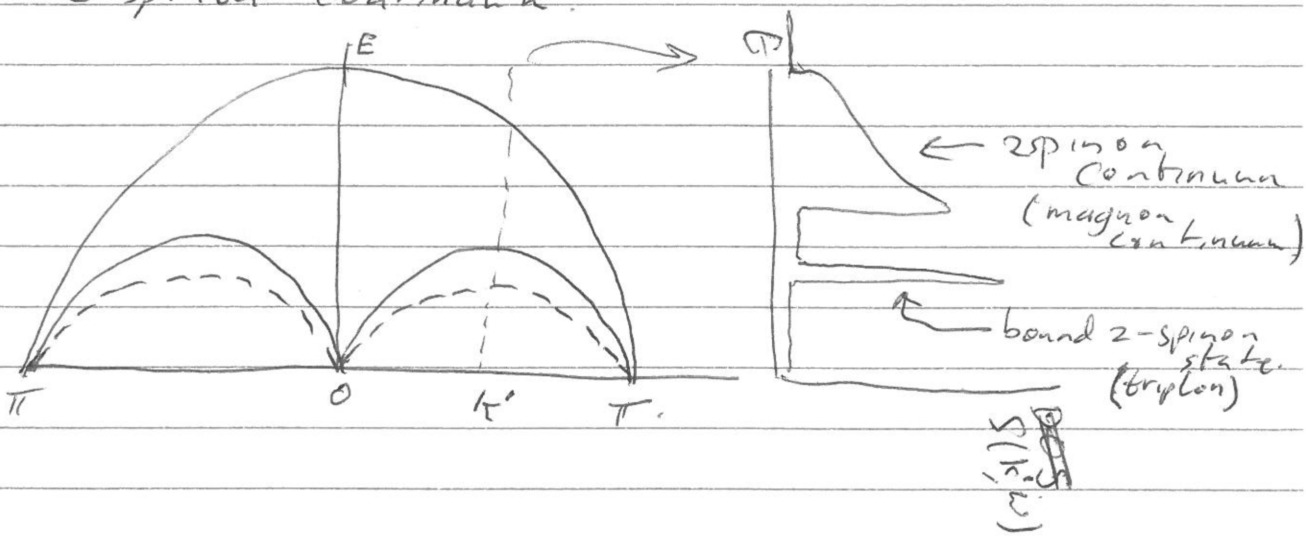
(Both J, J' are positive here!)

Then



'bound'.

energy bound ~~st~~ 2 spinon state lower than 2-spinon continuum.



$\boxed{\text{sheet } 3,4,5}$ Cs_2CuCl_4