

# *Magnetism*

---

**Paul H.M. van Loosdrecht**

*pvl@ph2.uni-koeln.de*

*Website:*

*www.loosdrecht.net*

**Optical Condensed Matter Physics**

# *Environment: crystal field*

---

Rare earth's: 4f shell's small ('inner' electrons)

Iron group: 3d shell's on the outside

=> decoupling of L and S, J no longer good Quantum number

=> splitting of the  $2L+1$  orbital states

=> Quenching of the orbital angular momentum ( $L_z \rightarrow 0$ )

=> High spin – Low spin transitions

=> Jahn-Teller distortions

=> Orbital excitations (orbitons)

Anisotropy

Kramers degeneracy (local B-field probe)

# Crystal field

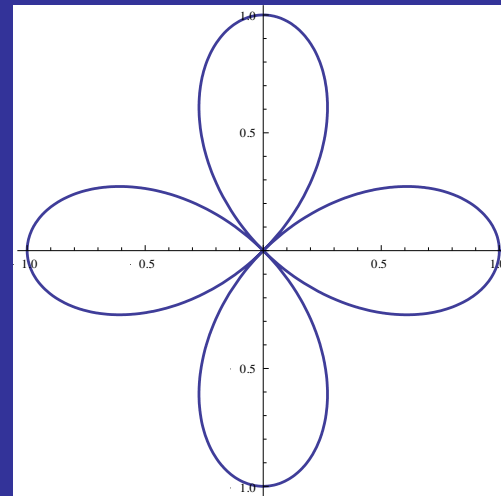
Simple illustrative example:  
2 dimensional p states in a two-fold crystal potential

p-states in 2D:

$$Y_{l,m}(\theta, \phi) = \cos(\theta) e^{im\phi} = e^{\pm i\phi}$$

$$p_{\pm 1, \sigma} = R(r) e^{\pm i\phi} \chi_{\sigma} \quad \sigma = \uparrow \text{ or } \downarrow$$

$$V_{CF} = Q \cos(2\phi)$$



# 2D $p$ -states in a $2_z$ -potential

Only CF  
 $V_{CF} = Q \cos(2\phi)$

$$\langle p_{1,\sigma} | V_{CF} | p_{1,\sigma} \rangle = \int d\phi e^{-i\phi} \cdot Q \cos(2\phi) \cdot e^{i\phi} = 0$$

$$\langle p_{-1,\sigma} | V_{CF} | p_{-1,\sigma} \rangle = \int d\phi e^{i\phi} \cdot Q \cos(2\phi) \cdot e^{-i\phi} = 0$$

$$\langle p_{-1,\sigma} | V_{CF} | p_{1,\sigma} \rangle = \int d\phi e^{i\phi} \cdot Q \cos(2\phi) \cdot e^{i\phi} = Q$$

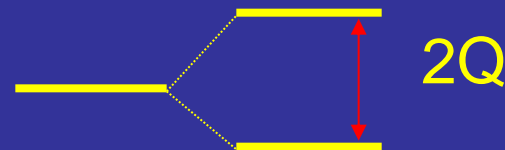
$$\langle p_{1,\sigma} | V_{CF} | p_{-1,\sigma} \rangle = \int d\phi e^{-i\phi} \cdot Q \cos(2\phi) \cdot e^{-i\phi} = Q$$

$$H = H_0 + V_{CF}$$

$$\begin{pmatrix} E_0 & Q & 0 & 0 \\ Q & E_0 & 0 & 0 \\ 0 & 0 & E_0 & Q \\ 0 & 0 & Q & E_0 \end{pmatrix} \cdot \begin{pmatrix} |p_{1,\uparrow}\rangle \\ |p_{-1,\uparrow}\rangle \\ |p_{1,\downarrow}\rangle \\ |p_{-1,\downarrow}\rangle \end{pmatrix} = E \cdot \begin{pmatrix} |p_{1,\uparrow}\rangle \\ |p_{-1,\uparrow}\rangle \\ |p_{1,\downarrow}\rangle \\ |p_{-1,\downarrow}\rangle \end{pmatrix}$$

$$E = E_0 \pm Q \quad +: |p_{1\sigma}\rangle + |p_{-1\sigma}\rangle$$

$$-: |p_{1\sigma}\rangle - |p_{-1\sigma}\rangle$$



# 2D p-states in a $2_z$ -potential

With LS  $\langle \Psi | \vec{L} \cdot \vec{S} | \Psi \rangle = \frac{\{J(J+1) - L(L+1) - S(S+1)\}}{2}$

$$\langle p_{1,\uparrow} | \vec{L} \cdot \vec{S} | p_{1,\uparrow} \rangle = \frac{\frac{15}{4} - 2 - \frac{3}{4}}{2} = \frac{1}{2}$$

$$\langle p_{-1,\uparrow} | \vec{L} \cdot \vec{S} | p_{-1,\uparrow} \rangle = \frac{\frac{3}{4} - 2 - \frac{3}{4}}{2} = -1$$

$$\langle p_{1,\downarrow} | \vec{L} \cdot \vec{S} | p_{1,\downarrow} \rangle = -1$$

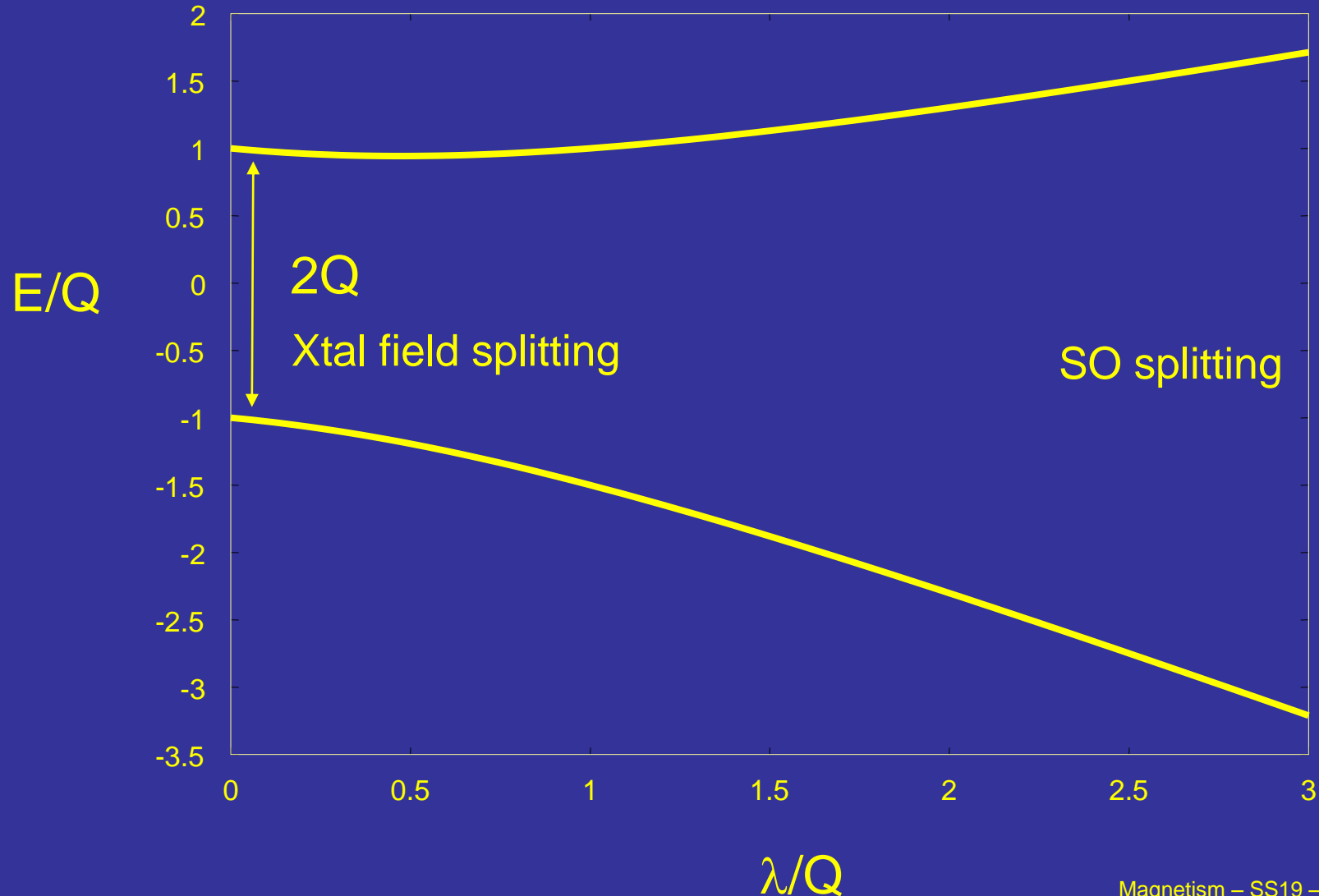
$$\langle p_{-1,\downarrow} | \vec{L} \cdot \vec{S} | p_{-1,\downarrow} \rangle = \frac{1}{2}$$

$$H = H_0 + V_{CF} + \lambda \vec{L} \cdot \vec{S} \quad \begin{pmatrix} \lambda/2 & Q & 0 & 0 \\ Q & -\lambda & 0 & 0 \\ 0 & 0 & \lambda/2 & Q \\ 0 & 0 & Q & -\lambda \end{pmatrix} \cdot \begin{pmatrix} |p_{1\uparrow}\rangle \\ |p_{-1\uparrow}\rangle \\ |p_{-1\downarrow}\rangle \\ |p_{1\downarrow}\rangle \end{pmatrix} = E \cdot \begin{pmatrix} |p_{1\uparrow}\rangle \\ |p_{-1\uparrow}\rangle \\ |p_{-1\downarrow}\rangle \\ |p_{1\downarrow}\rangle \end{pmatrix}$$

$$E_{\pm} = -\lambda/4 \pm \sqrt{9\lambda^2/16 + Q^2} \quad \begin{aligned} + &: u|p_{1\sigma}\rangle + v|p_{-1\sigma}\rangle \\ - &: v|p_{1\sigma}\rangle - u|p_{-1\sigma}\rangle \end{aligned} \quad \begin{aligned} u^2 &= \frac{1}{2} + \frac{3\lambda/8}{\sqrt{9\lambda^2/16 + Q^2}} \\ v^2 &= \frac{1}{2} - \frac{3\lambda/8}{\sqrt{9\lambda^2/16 + Q^2}} \end{aligned}$$

# 2D p-states in a $2_z$ -potential

Energy of states as a function of LS/CF



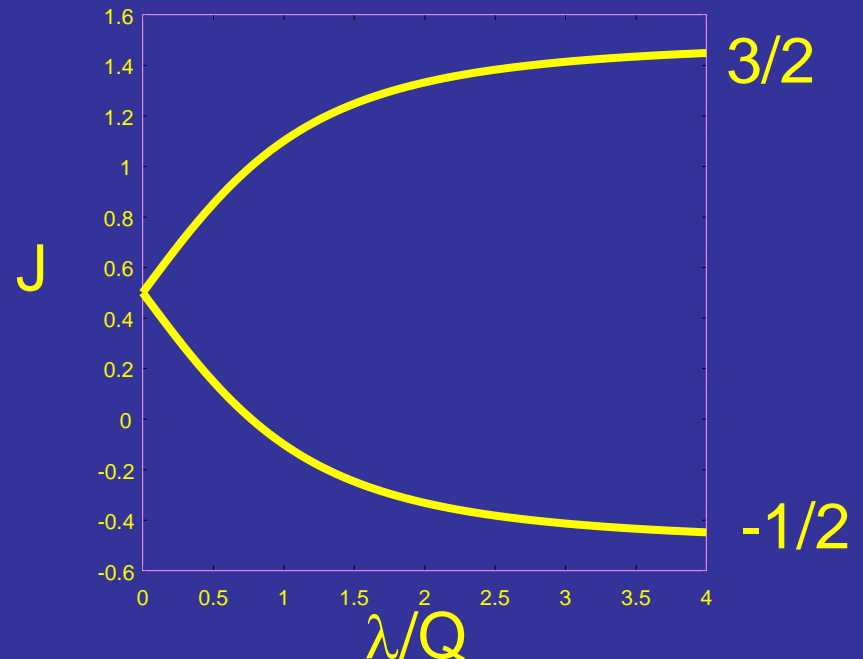
# 2D p-states in a $2_z$ -potential

Angular momentum of the states as a function of LS/CF

$$\begin{aligned} \langle \phi_{+, \uparrow} | \mathbf{J} | \phi_{+, \uparrow} \rangle &= u^2 \cdot \langle p_{1, \uparrow} | \mathbf{J} | p_{1, \uparrow} \rangle + v^2 \cdot \langle p_{-1, \uparrow} | \mathbf{J} | p_{-1, \uparrow} \rangle \\ &= \left\{ \frac{1}{2} + \frac{3\lambda/8}{\sqrt{9\lambda^2/16 + Q^2}} \right\} \cdot \frac{3}{2} + \left\{ \frac{1}{2} - \frac{3\lambda/8}{\sqrt{9\lambda^2/16 + Q^2}} \right\} \cdot \frac{-1}{2} \end{aligned}$$

$$= \frac{1}{2} + \frac{3\lambda/4}{\sqrt{9\lambda^2/16 + Q^2}}$$

$$\langle \phi_{+, \downarrow} | \mathbf{J} | \phi_{+, \downarrow} \rangle = \frac{1}{2} - \frac{3\lambda/4}{\sqrt{9\lambda^2/16 + Q^2}}$$



For strong crystal fields:  
orbital moment completely quenched

# 2D $p$ -states in a $2_z$ -potential

Magnetic moment of the states as a function of LS/CF

$$\langle p_{1,\uparrow} | m_z | p_{1,\uparrow} \rangle = (L_z + gS_z) \mu_B = (1 + 2 \cdot \frac{1}{2}) \mu_B = 2 \mu_B$$

$$\langle p_{1,\downarrow} | m_z | p_{1,\downarrow} \rangle = (L_z - gS_z) \mu_B = (1 - 2 \cdot \frac{1}{2}) \mu_B = 0$$



$$\langle \phi_{+,\uparrow} | m_z | \phi_{+,\uparrow} \rangle = u^2 \cdot \langle p_{1,\uparrow} | m_z | p_{1,\uparrow} \rangle + v^2 \cdot \langle p_{-1,\uparrow} | m_z | p_{-1,\uparrow} \rangle = \left\{ 1 + \frac{3\lambda/4}{\sqrt{9\lambda^2/16 + Q^2}} \right\} \cdot \mu_B$$

$$\langle \phi_{+,\downarrow} | m_z | \phi_{+,\downarrow} \rangle = \left\{ 1 - \frac{3\lambda/4}{\sqrt{9\lambda^2/16 + Q^2}} \right\} \cdot \mu_B$$

