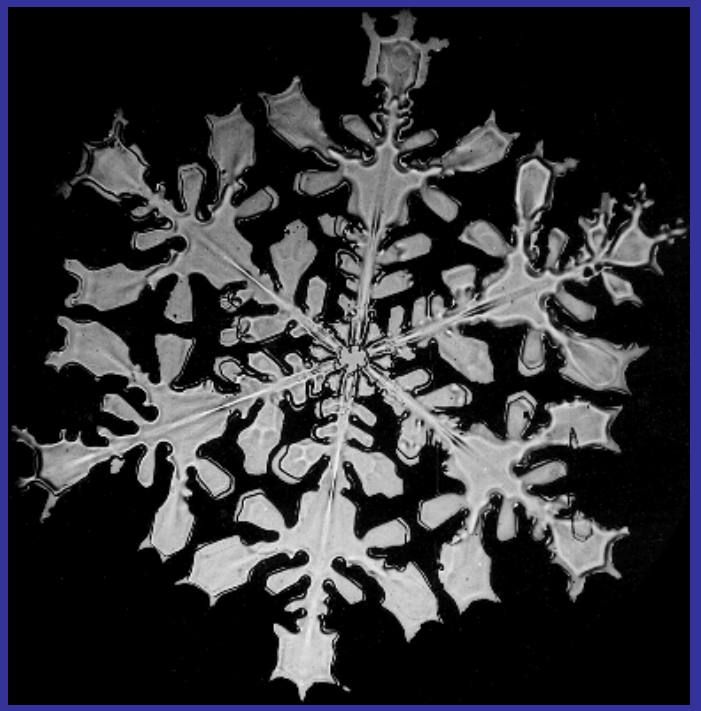
Symmetry

BROKEN SYMMETRY

- Phase transitions (critical behaviour, 1st/2nd order)
- Rigidity (crystals don't bend, permanent M)
- Excitations (spectral changes, soft mode, order parameter)
- Defects (order parameter discontinuity)



What is the difference between phases ?

Density, correlations, symmetry
color, magnetisation, charge state, etc

Symmetry breaking transitions (solid-liquid)

Liquid: High symmetry
Solid: Few symm. elements kept.

Non-breaking transitions (gas-liquid)

Density

Universality classes depending on

- d, dimension of space
- D, dimension of order parameter
- short/long range order

Some examples

Liquid:

Each positions in space occupied (t-average)

High symmetry (all rotations & translations)

Solid:

Few positions occupied

Low symmetry (few rotations & translations)

C_{60}:

High T: FCC

Low T: SC

Magnetic:

High T: No preferred spin direction

Low T : Alignment

Landau theory

ORDER PARAMETER:

Density, Position, Dipole moment,
Magnetization, Occupation number,
Orientation, …

FREE ENERGY

$F = E – T \cdot S$

E: energy, S: entropy ($S = k_b \ln(\#\ \text{states})$)

LANDAU THEORY

- Free energy order parameter expansion
- Linearize near T_c
- Minimize free energy
- Mean field approach:
  correlations or fluctuations not included

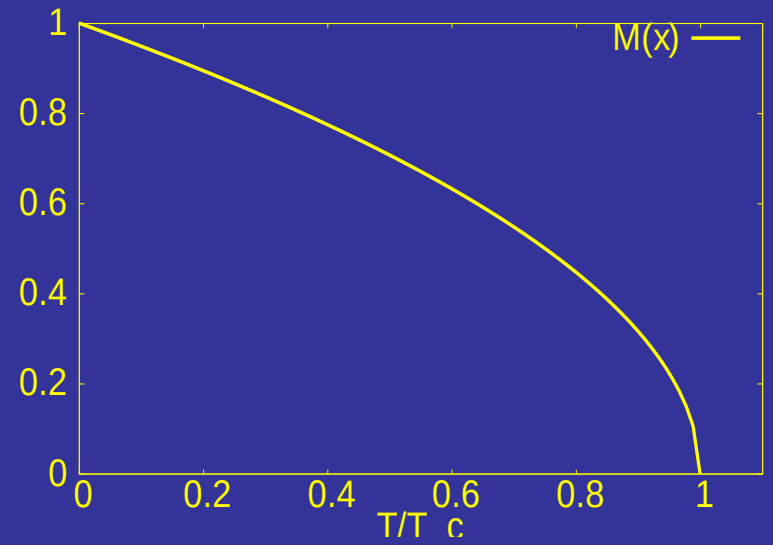
Example: Ferromagnet, order parameter: M

Landau theory

$$F(M) = F_0 + a_0\left(T - T_c\right)M^2 + bM^4$$

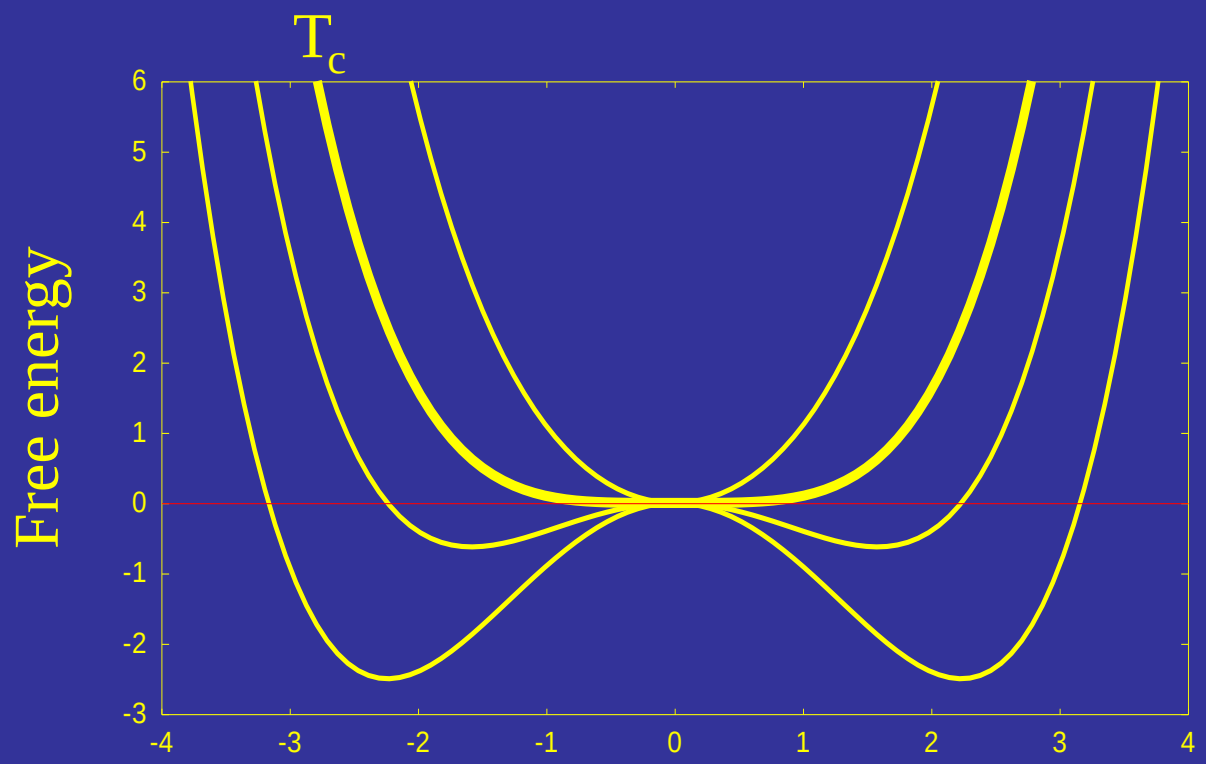
$$T < T_c : M = \pm\sqrt{\frac{a_0(T_c - T)}{2b}}$$

$$T_c = \frac{2b}{a_0}M_{\max}^2$$



Landau theory

Degenerate ground state
Two metastable states



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