

Amplitude reflection/transmission coefficients for normal incidence coming from a medium with  $n=1$  (vacuum), going into a medium with  $n$ .

$$r' = \frac{n-1}{n+1}; \quad t = \frac{2}{n+1}$$

For the reverse situation:

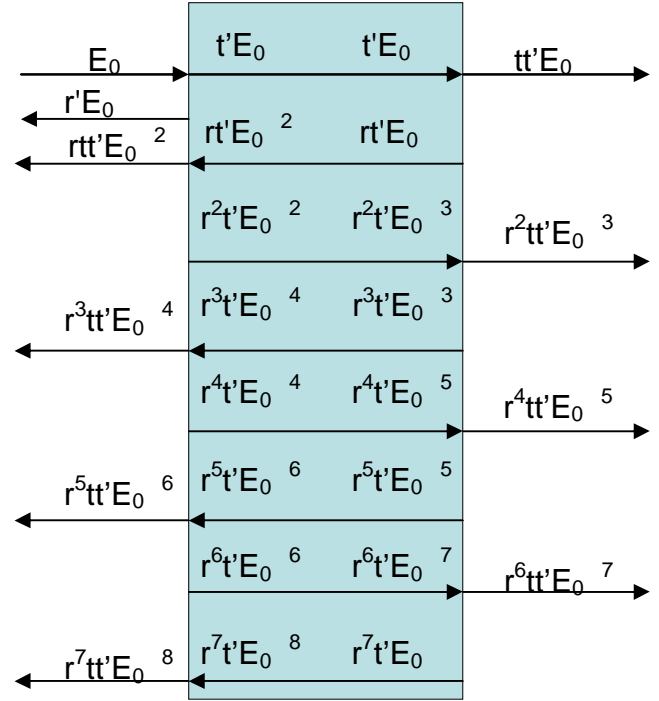
$$r = \frac{1-n}{n+1} = -r'; \quad t = \frac{2 \cdot n}{n+1} = n \cdot t'$$

For normal incidence we thus have:

$$r + t = 1; \quad r' + t' = 1$$

Further (no absorption):  $R = r^2 = r'^2$   
and  $T = t \cdot t'$ ;  $R + T = 1$ .

So for the total amplitude reflection we have  
( $\delta = \exp(-i \cdot \Delta / 2)$ , with  $\Delta = 2 \cdot k \cdot d$  the round trip phase difference,  $k = 2\pi / \lambda$ ,  $d$  thickness of the slab):



$$r'_{\text{fp}} = \frac{E_r}{E_0} = -r' + t \cdot t' \sum_{k=1}^{\infty} r'^{2 \cdot k-1} \cdot \delta^{2 \cdot k} = \frac{r' \cdot (r'^2 \delta^2 - 1 + t \cdot t' \cdot \delta^2)}{1 - r'^2 \delta^2} = \frac{r' \cdot (\delta^2 - 1)}{1 - R} \cdot \frac{(1 - R)}{1 - R \delta^2}$$

For the field in the cavity:

$$w_{\text{fp}} = \frac{E_w}{E_0} = t \cdot \left( \sum_{k=0}^{\infty} r'^{2 \cdot k} \cdot \delta^{2 \cdot k} + \sum_{k=1}^{\infty} r'^{2 \cdot k-1} \cdot \delta^{2 \cdot k} \right) = \frac{t \cdot (1 + R \delta^2)}{1 - r'^2 \delta^2} = \frac{1}{n} \cdot \frac{(1 - r') \cdot (1 + r' \cdot \delta^2)}{(1 - R)} \cdot \frac{(1 - R)}{1 - R \delta^2}$$

For the transmitted field:

$$t_{\text{fp}} = \frac{E_t}{E_0} = t \cdot t' \cdot \delta \cdot \left( \sum_{k=0}^{\infty} r'^{2 \cdot k} \cdot \delta^{2 \cdot k} \right) = \frac{t \cdot t' \cdot \delta}{1 - r'^2 \delta^2} = \delta \cdot \frac{1 - R}{1 - R \delta^2}$$

The intensities are given by:

$$R_{\text{fp}} = r_{\text{fp}} \cdot r_{\text{fp}}^*; \quad W_{\text{fp}} = n \cdot w_{\text{fp}} \cdot w_{\text{fp}}^*; \quad T_{\text{fp}} = t_{\text{fp}} \cdot t_{\text{fp}}^*$$

which can be calculated using:  $\delta \cdot \delta^* = 1$ ;  $\delta^2 + \delta^{*2} = 2 \cdot \cos(\Delta) = 2 \cdot (1 - 2 \cdot \sin^2(\Delta / 2))$  and defining the coefficient of

finesse:  $F = \frac{4R}{(1-R)^2}$ . The answers are:

$$R_{\text{fp}} = \frac{F \sin^2(\Delta / 2)}{1 + F \sin^2(\Delta / 2)}; \quad T_{\text{fp}} = \frac{1}{1 + F \sin^2(\Delta / 2)}, \text{ as expected } R_{\text{fp}} + T_{\text{fp}} = 1, \text{ and}$$

$$W_{\text{fp}} = \frac{1 - \frac{4 \cdot \sqrt{R}}{1 + \sqrt{R}} \sin^2(\Delta / 2)}{1 + F \sin^2(\Delta / 2)}$$