Last time

- Dipole – Dipole
- Direct exchange (H_2 molecule)
- Indirect exchange
- Double exchange
- Anisotropic exchange
- Rudeman Kittel Kasuya Yoshida (RKKY)
- Stoner ("spontaneous Pauli")

- Ch. 4 & 7.2,7.3,7.7

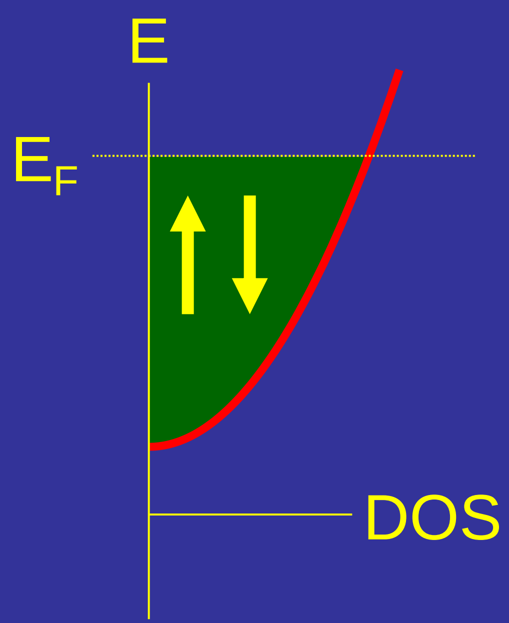
Lectures

Lect. 1	Introduction	Ch.1; 2.1-2.5; 8.9
Lect. 2	Interactions, environment	3.1, Ch.4; 7.1-7.7
Lect. 3	Ordering, Domains	5.1-5.3; 6.7; 8.3, 8.7, 8.8
Lect. 4	Symmetry breaking	6.1-6.6
Lect. 5	Quantum magnetism	8.1-8.6

Magnetism in metals

Free electron gas

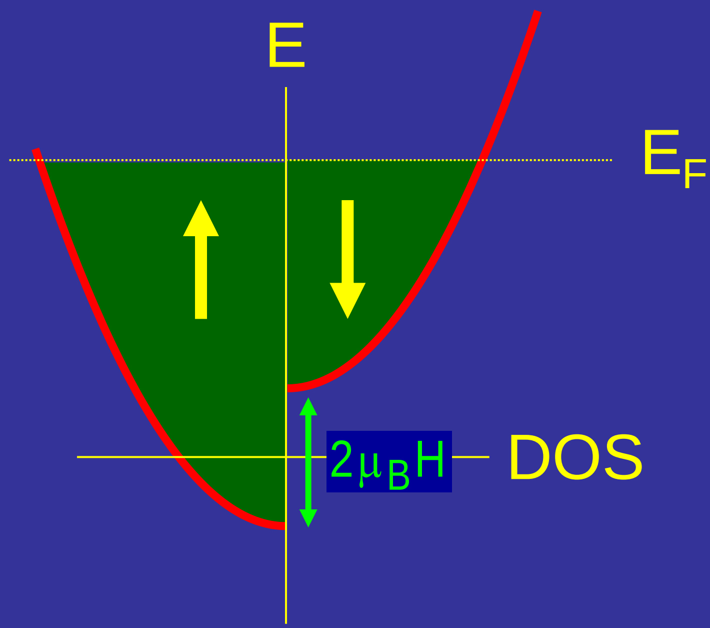
No field: $E = \dfrac{\hbar^2 k^2}{2m^*}$ $\quad E_F = \dfrac{\hbar^2}{2m^*}\left(3\pi^2 n\right)^{2/3}$ $\quad D(E) = \dfrac{1}{2\pi^2}\left(\dfrac{2m^*}{\hbar^2}\right)^{3/2}\sqrt{E}$



- Pauli paramagnetism, Landau diamagnetism
- Spontaneous spin polarization (Stoner)
- RKKY

Pauli paramagnetism

H ≠ 0 : $E = \frac{\hbar^2 k^2}{2m^*} \pm \frac{1}{2} g_o \mu_B H$



$$N_\uparrow = \frac{1}{2}\int_{-\mu_B}^{E_F} D(E+\mu_B H)\, dE$$

$$\approx \frac{1}{2}\left(\int_{0}^{E_F} D(E)\, dE + \mu_B H \cdot D(E_F)\right)$$

$$N_\downarrow \approx \frac{1}{2}\left(\int_{0}^{E_F} D(E)\, dE - \mu_B H \cdot D(E_F)\right)$$

Pauli: $M = \mu_B (N_\uparrow - N_\downarrow)$

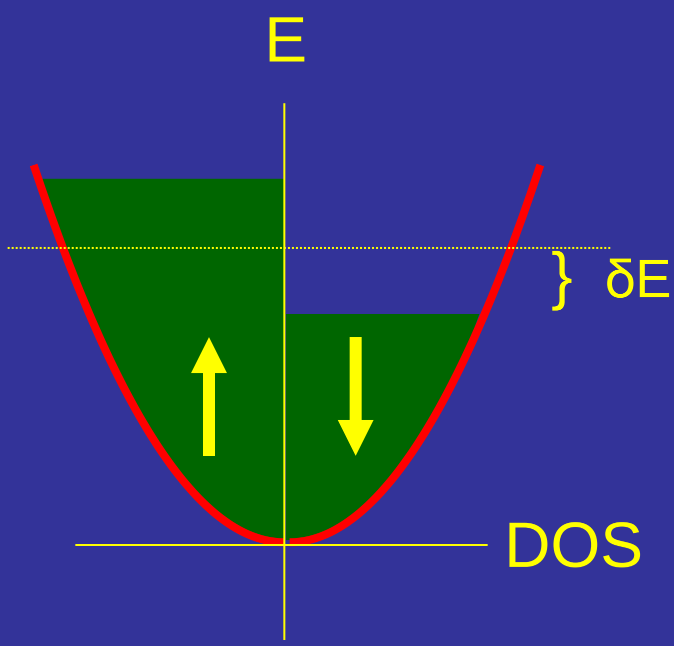
$$= \mu_B^2\, D(E_F)\, H = \frac{3n\mu_B^2}{2kT_F} H$$

Landau (dia): $M = -\frac{n\mu_B^2}{2kT_F} H$

$$\Rightarrow \boxed{\chi_e = \frac{n\mu_B^2}{kT_F}}$$

Stoner magnetism

Spontaneous spin polarization



If spin split then 'internal' field $H = \lambda M$

Cost in kinetic energy : $\Delta E_k = \left[\frac{1}{2} D(E_F) \cdot \delta E\right] \cdot \delta E$

Magnetization :

$$n_\uparrow = \frac{1}{2}\left(n + D(E_F) \cdot \delta E\right); \qquad n_\downarrow = \frac{1}{2}\left(n - D(E_F) \cdot \delta E\right)$$

$$M = \mu_B (n_\uparrow - n_\downarrow) = \mu_B D(E_F) \cdot \delta E$$

Field energy

$$\Delta E_p = -\int_0^M B dM' = -\int_0^M \mu_0 (\lambda M') dM' = -\frac{1}{2} \mu_0 \lambda M^2$$

$$= -\frac{1}{2} \mu_0 \mu_B^2 \lambda (n_\uparrow - n_\downarrow) = -\frac{1}{2} U (n_\uparrow - n_\downarrow) = -\frac{1}{2} U [D(E_F) \cdot \delta E]^2$$

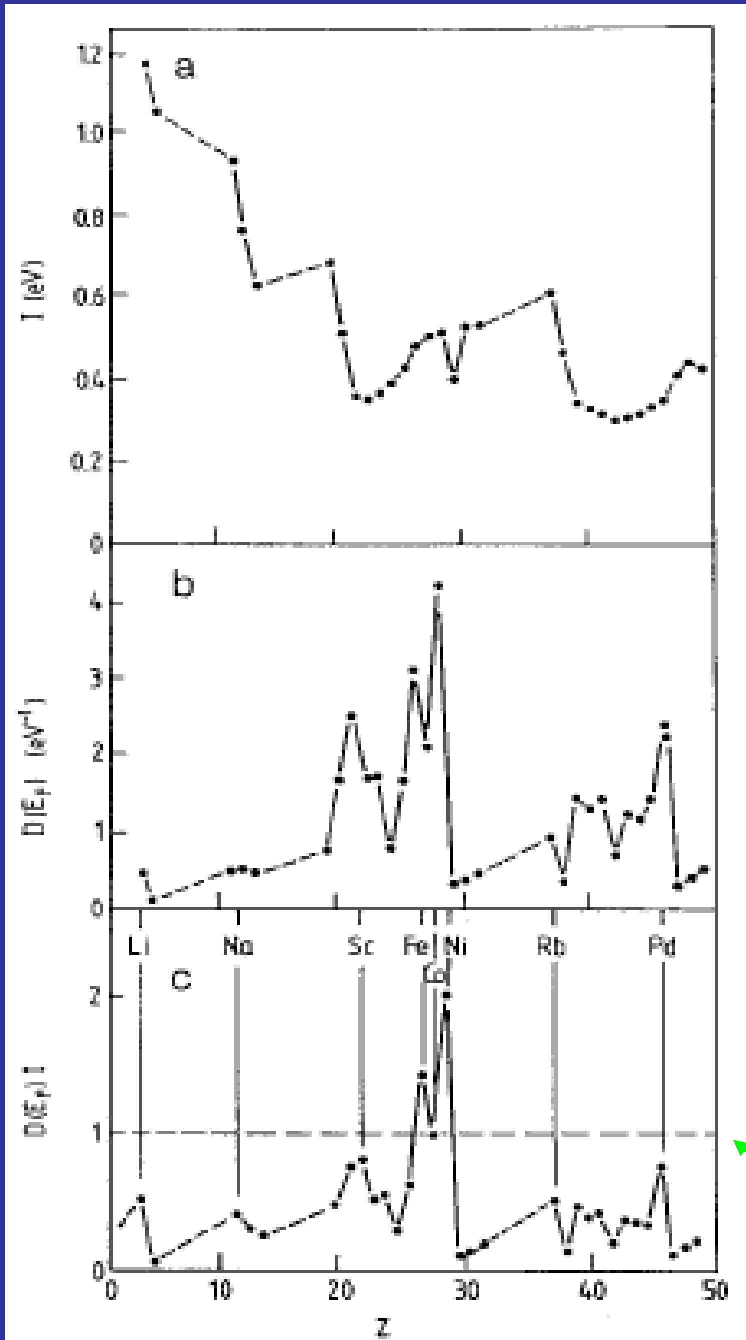
Total energy $\Delta E_{tot} = \Delta E_k + \Delta E_p$

$$\Delta E_{tot} = \frac{1}{2} D(E_F) \cdot \delta E^2 [1 - U D(E_F)]$$

If $[1 - U D(E_F)] < 0$ then $\Delta E_{tot} < 0 \Rightarrow$ Magnetic ground state

Happens for strong Coulomb and high D.O.S.

Stoner criterium



Exchange interaction

Density of States at E_F

Product

In agreement with
Fe, Ni, Co ferromagnets

Stoner criterium

Stoner magnetism

- If $UD(E_F)<1$ then still change in susceptibility

$$\text{Total energy in external field } B: \Delta \mathrm{E}_{\mathrm{tot}} = \Delta \mathrm{E}_{\mathrm{k}} + \Delta \mathrm{E}_{\mathrm{p}} - MB$$

$$\mathrm{M} = \mu_{\mathrm{B}}(n_{\uparrow} - n_{\downarrow}) = \mu_{\mathrm{B}} D(E_F) \cdot \delta E$$

$$\Delta \mathrm{E}_{\mathrm{tot}} = \frac{1}{2} \frac{M^2}{\mu_B^2 D(E_F)} [1 - UD(E_F)] - M \cdot B$$

Minimization w.r.t. M leads to

$$M = \frac{\mu_B^2 D(E_F)}{[1 - UD(E_F)]} B$$

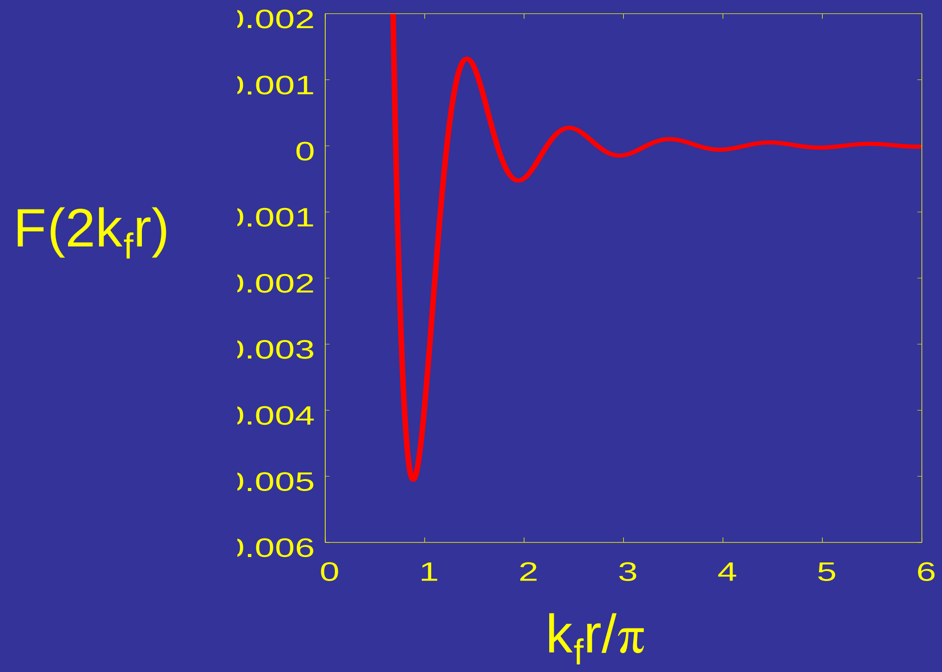
$$\text{Susceptibility } \chi = \mathrm{M/H} = \mathrm{M}/(B/\mu_0)$$

$$\chi = \frac{\mu_0 \mu_B^2 D(E_F)}{[1 - UD(E_F)]} = \frac{\chi_{Pauli}}{[1 - UD(E_F)]}$$

$\Rightarrow$ Enhanced susceptibility

Spatially varying fields

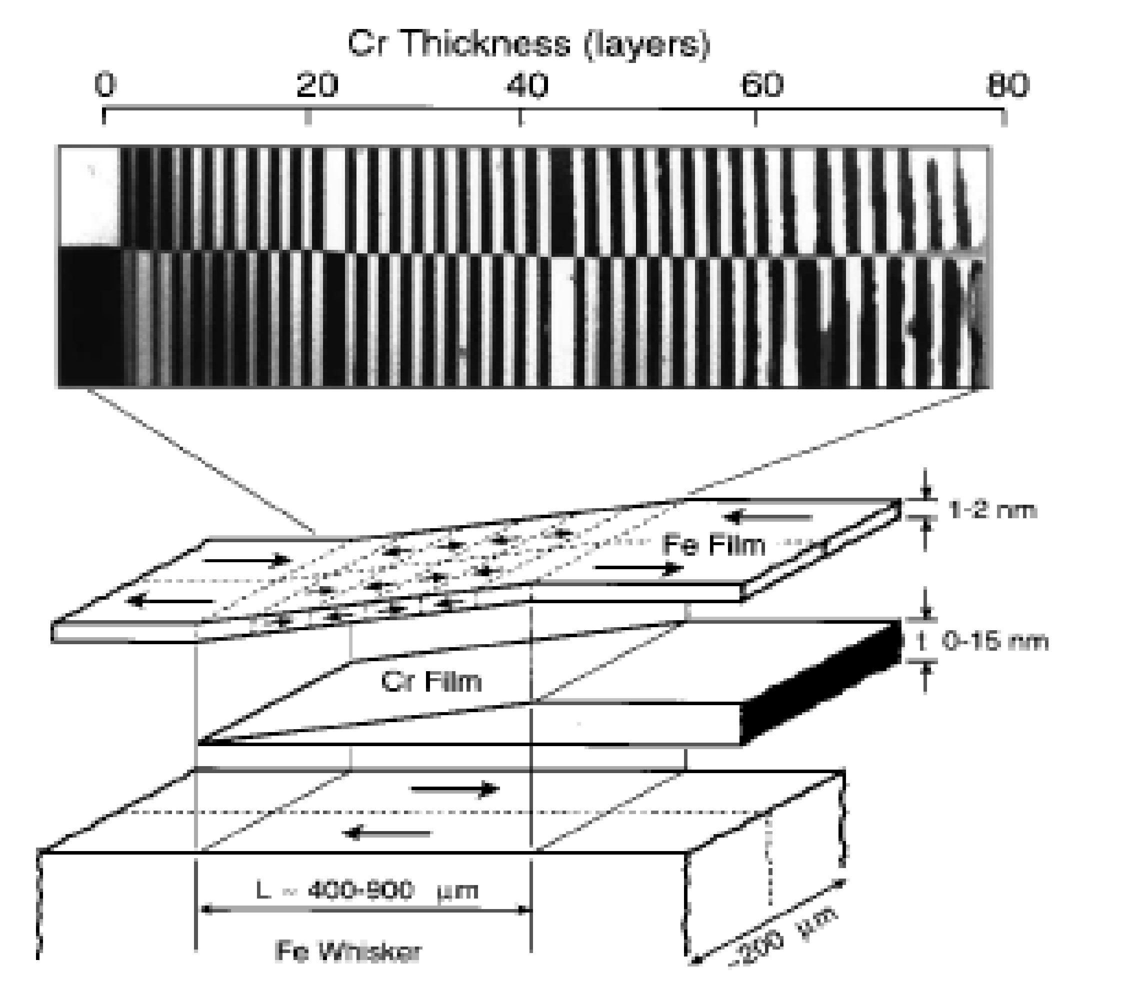
- RKKY interaction *(Ruderman-Kittel-Kasuya-Yosida)* (par. 7.7)

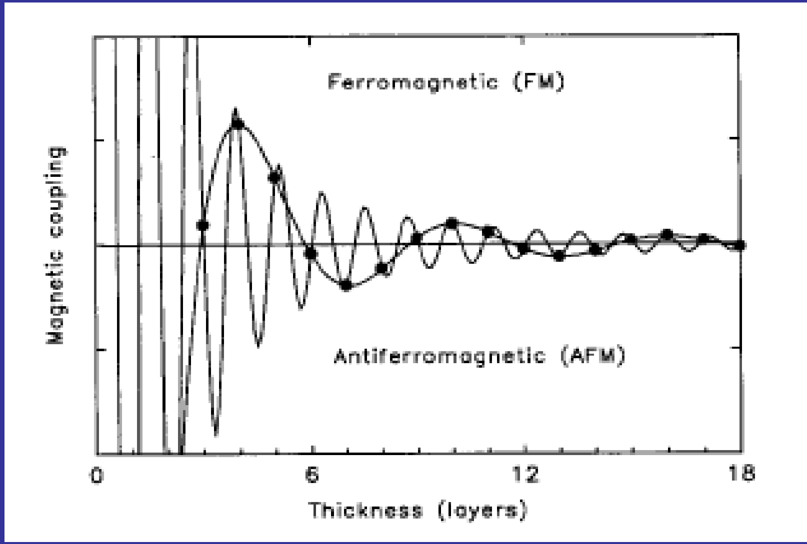


$$H(r) = H\ \delta(r)$$

$$\chi(r) = \frac{2}{\pi} k_f^3 \chi_{pauli} F(2k_f r) \overset{x>>1}{=} -\frac{2}{\pi} k_f^3 \chi_{pauli} \frac{\cos(2k_f r)}{(2k_f r)^3}$$

RKKY interaction





RKKY interaction

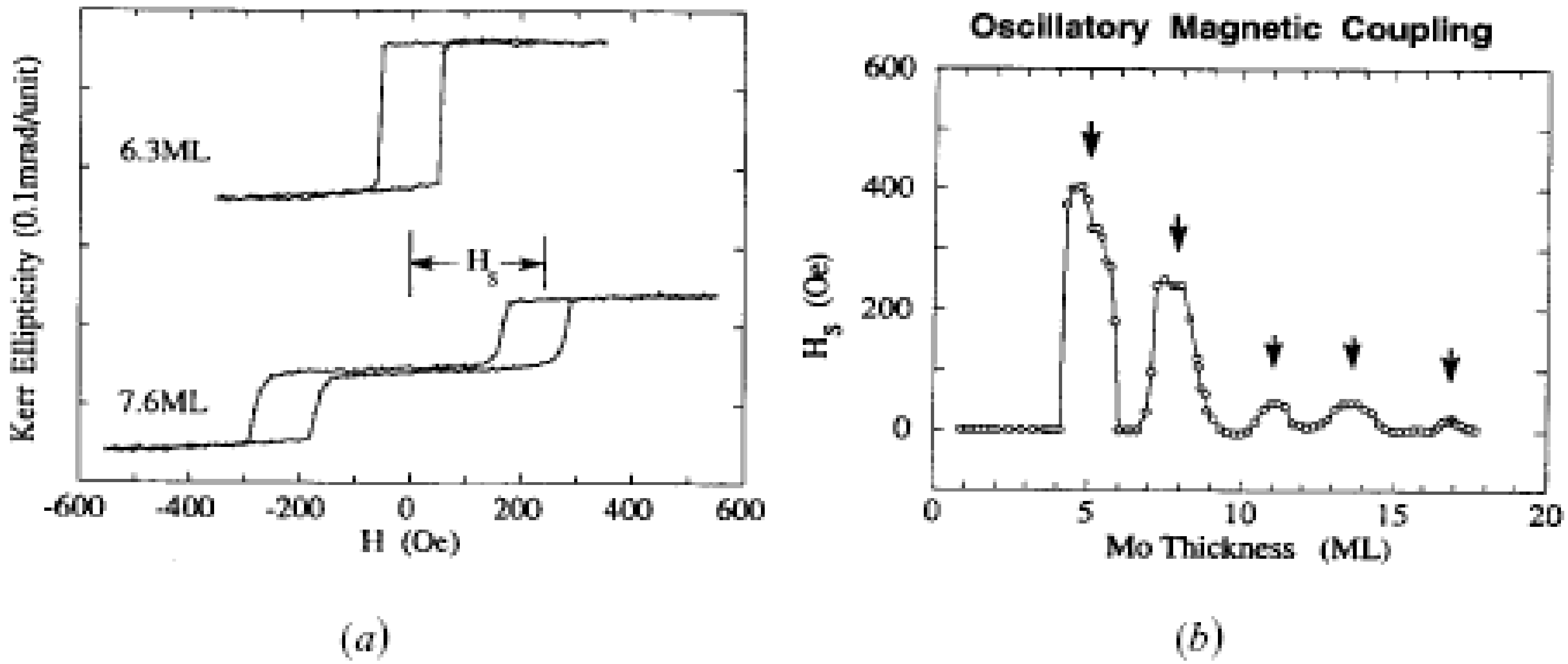


Figure 38. Magnetic oscillations at Fe/Mo/Fe(100) trilayers determined by the SMOKE (Qiu *et al.* 1992b). (*a*) Hysteresis loops characteristic of parallel and antiparallel coupling (top and bottom). H_s is the magnetic field required to force antiparallel layers parallel. Adding just slightly more than a monolayer to the Mo spacer reverses the magnetic orientation. (*b*) Alternating antiparallel and parallel coupling (arrows and baseline respectively).

Ch. 5 Ordered magnetism

- Ferromagnets
- Antiferromagnets
- Ferrimagnets
- Helical order

- Weiss model (Mean field, ferromagnetic, L=0)

$$H = -\sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j + g\mu_B \sum_i \vec{S}_i \cdot \vec{B}$$

Mean field model

- Each spin feels the field of all other

$$\boxed{\text{Field of other spins}: \vec{H}_{mf} = \lambda \vec{M}}$$

$$E_i \overset{m.f.}{=} -2\sum_j J_{ij}\vec{S}_i \cdot \left\langle \vec{S}_j \right\rangle \overset{n.n.}{=} -2Jz\left\langle \vec{S} \right\rangle \cdot \vec{S}_i \equiv \vec{m}_i \cdot \vec{H}_{mf} = -g\mu_B \vec{S}_i \cdot \vec{H}_{mf}$$

$$\left.\begin{aligned} \vec{H}_{mf} &= \frac{2Jz}{g\mu_B}\left\langle \vec{S} \right\rangle := \lambda \vec{M} \\ \left\langle \vec{S} \right\rangle &= \frac{\vec{M}}{ng\mu_B} \end{aligned}\right\} \lambda = \frac{2Jz}{(g\mu_B)^2 n}$$

Above ordering temperature

Paramagnetic → $M = \chi_{curie} H_{total} = \frac{C}{T} H_{total}$

$$C = \frac{n\left(p\mu_B\right)^2}{3k}$$

$$p = g_J \sqrt{J\left(J+1\right)}$$

Field of other spins : $H_{mf} = \lambda M$

Total field on a spin : $H_{total} = H_{ext} + H_{mf}$

Magnetization : $M = \chi_{curie} H_{total} = \frac{C}{T}\left(H_{ext} + H_{mf}\right) = \frac{C}{T}\left(H_{ext} + \lambda M\right)$

$$\frac{M}{H_{ext}} \equiv \chi = \frac{C/T}{1-\lambda \frac{C}{T}} = \frac{C}{T-\lambda C} = \frac{C}{T-T_C}$$

T_C : Curie - Weiss temperature

$$\left.\begin{aligned} T_C &= \lambda C \\ C &= \frac{n\left(p\mu_B\right)^2}{3k} \\ \lambda &= \frac{2Jz}{\left(g\mu_B\right)^2 n} \end{aligned}\right\} \quad T_c = \frac{Jzp^2}{6k} \qquad J = \frac{6}{zp^2} kT_c$$

Below T_C: Sponteneous order

Spontaneous order ➔ $H_{ext} = 0; \quad M \neq 0; \quad H_{mf} = \lambda M$

$$\text{Field of other spins}: H_{mf} = \lambda M$$

$$\text{Total field on a spin}: H_{total} = H_{mf}$$

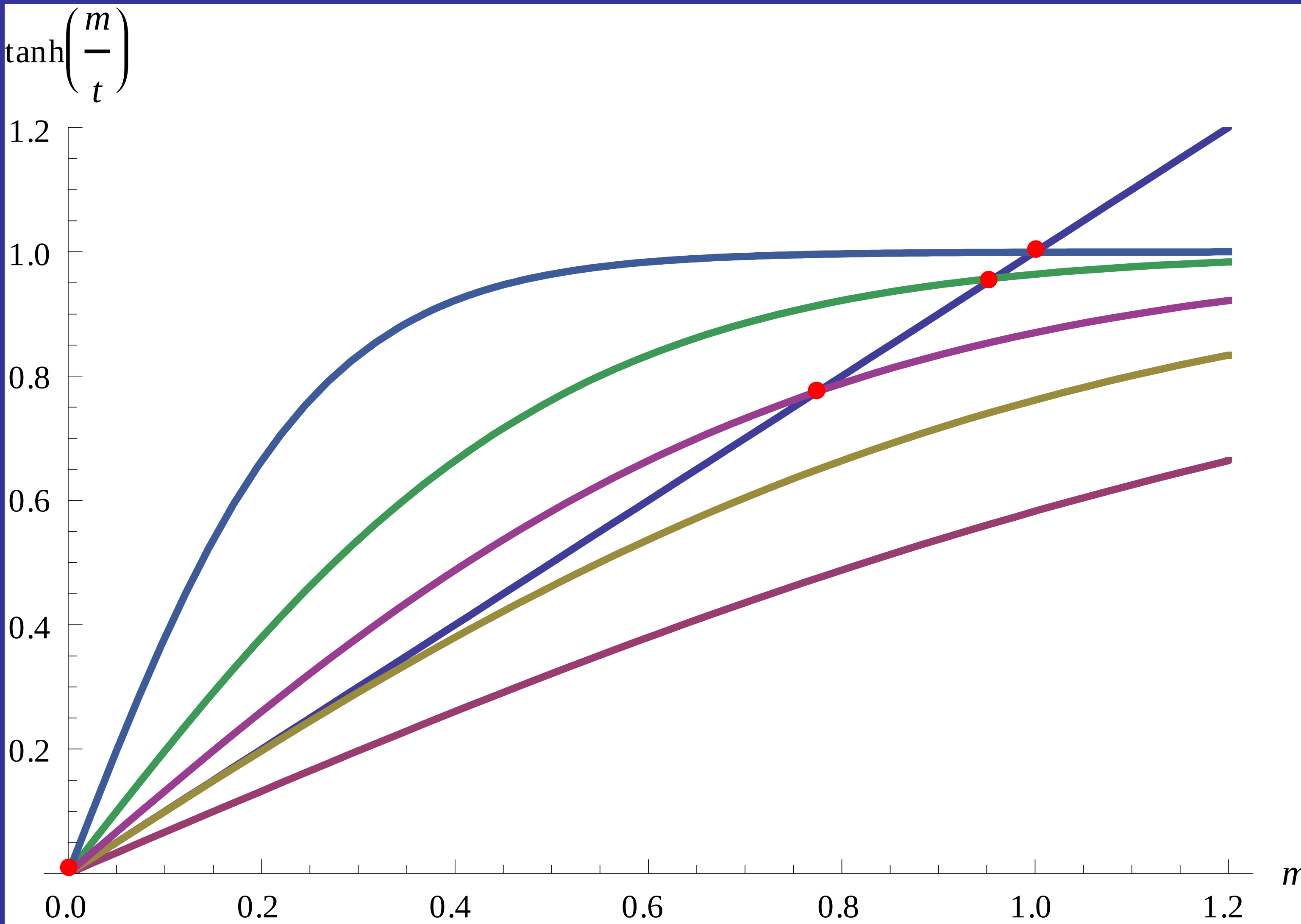
$$\text{Magnetization}: M = n g_J \mu_B J B_J(x) \text{ with } x = \frac{g_J \mu_B J H_{mf}}{kT} = \frac{g_J \mu_B J \lambda M}{kT}$$

$$\text{For } g = 2; J = 1/2: \quad M = n\mu_B \tanh\left(\frac{\mu_B \lambda M}{kT}\right)$$

$$\text{or } m = \tanh(m/t)$$

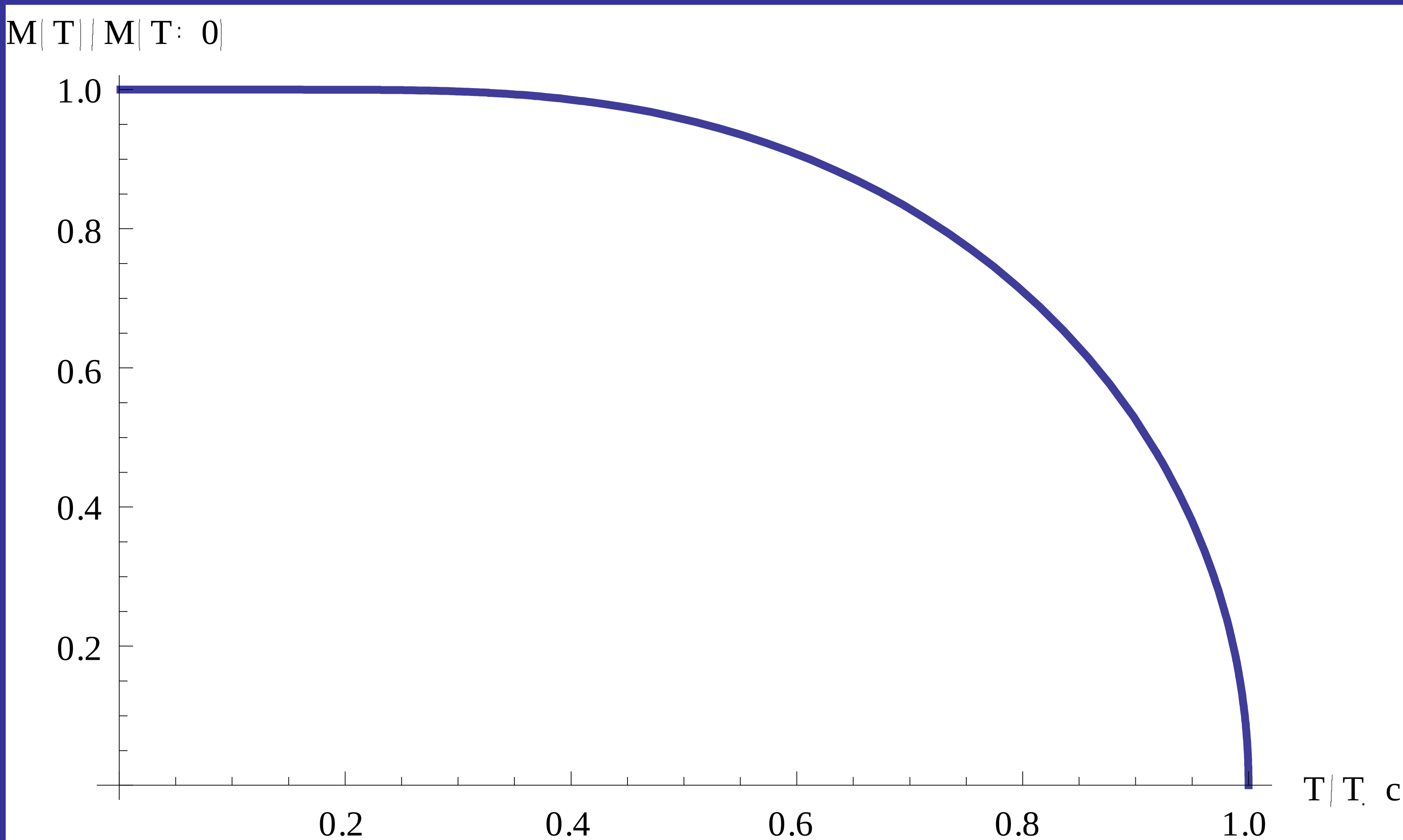
$$\text{with } m = \frac{M}{n\mu_B} \text{ and } t = \frac{kT}{\lambda n \mu_B^2}$$

Below T_C: Sponteneous order



t
0.25
0.5
0.75
1
1.5

Below T_C: Sponteneous order



Numerically solved m=tanh(m/t)

Ferromagnets

	z	n [10^{22} cm^{-3}]	g	p	C [K]	T_c [K]	J [meV]	λ
Fe	8	8.5	2	5.4	0.51	1043	2.3	2045
Co	12	9	2	4.8	0.43	1388	2.3	3228
Ni	12	9.1	2	3.2	0.19	627	0.6	3300
Gd	12	3	2	8.0	0.40	293	0.2	733

	M(0) [gauss]	M(0)/Nμ_B	H_{mf} [10^6 gauss]
Fe	1740	2.22	3.6
Co	1446	1.72	4.7
Ni	510	0.606	1.7
Gd	2060	7.63	1.5

Excercises

- 3.1; 3.5 ; 3.11
- Which 3d ions can be Jahn Teller active, what are the corresponding d-electron occupations ?
- The lowest 3d levels in an octahedral field are the t_{2g} triplet (states like d_{xy}). Can you predict the energy splitting in a tetrahedral environment
- 4.4; 4.5; 5.1; 6.13