Condensed Matter Physics I

Prof. Dr. Ir. Paul H.M. van Loosdrecht

II Physikalisches Institut, Room 312

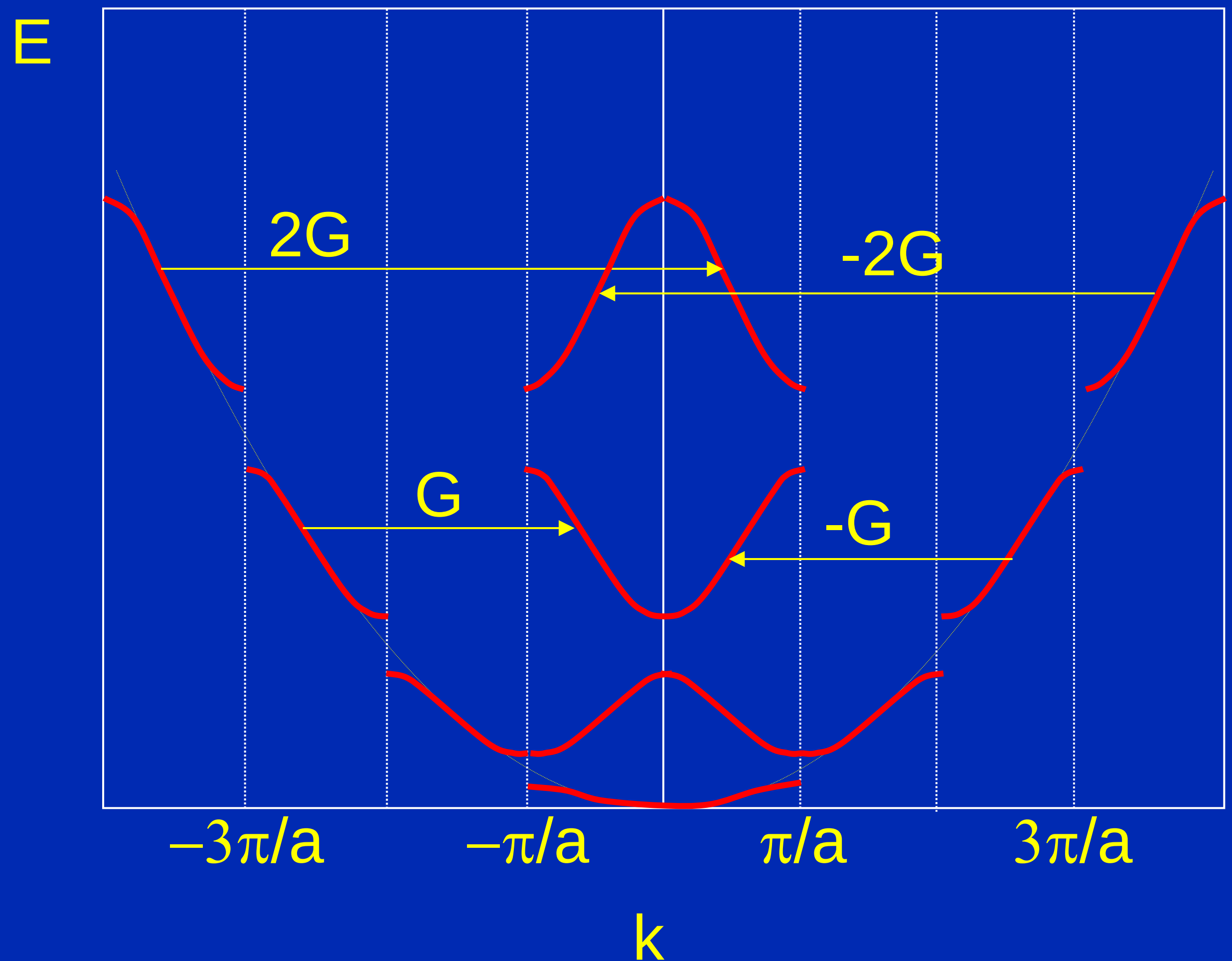
E-mail: pvl@ph2.uni-koeln.de

Website: http:/www.loosdrecht.net/

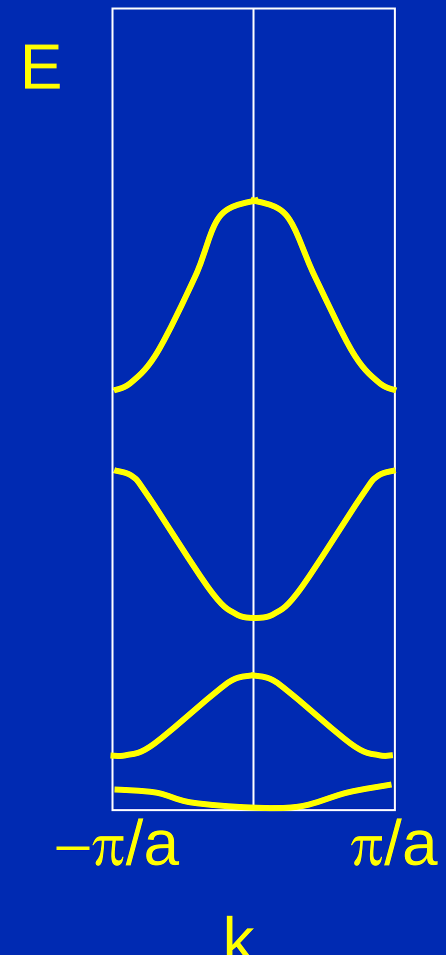
Today

- Fermi surfaces (Kittel Ch.9)
- Semiconductors (Kittel Ch.8)

Extended zone scheme

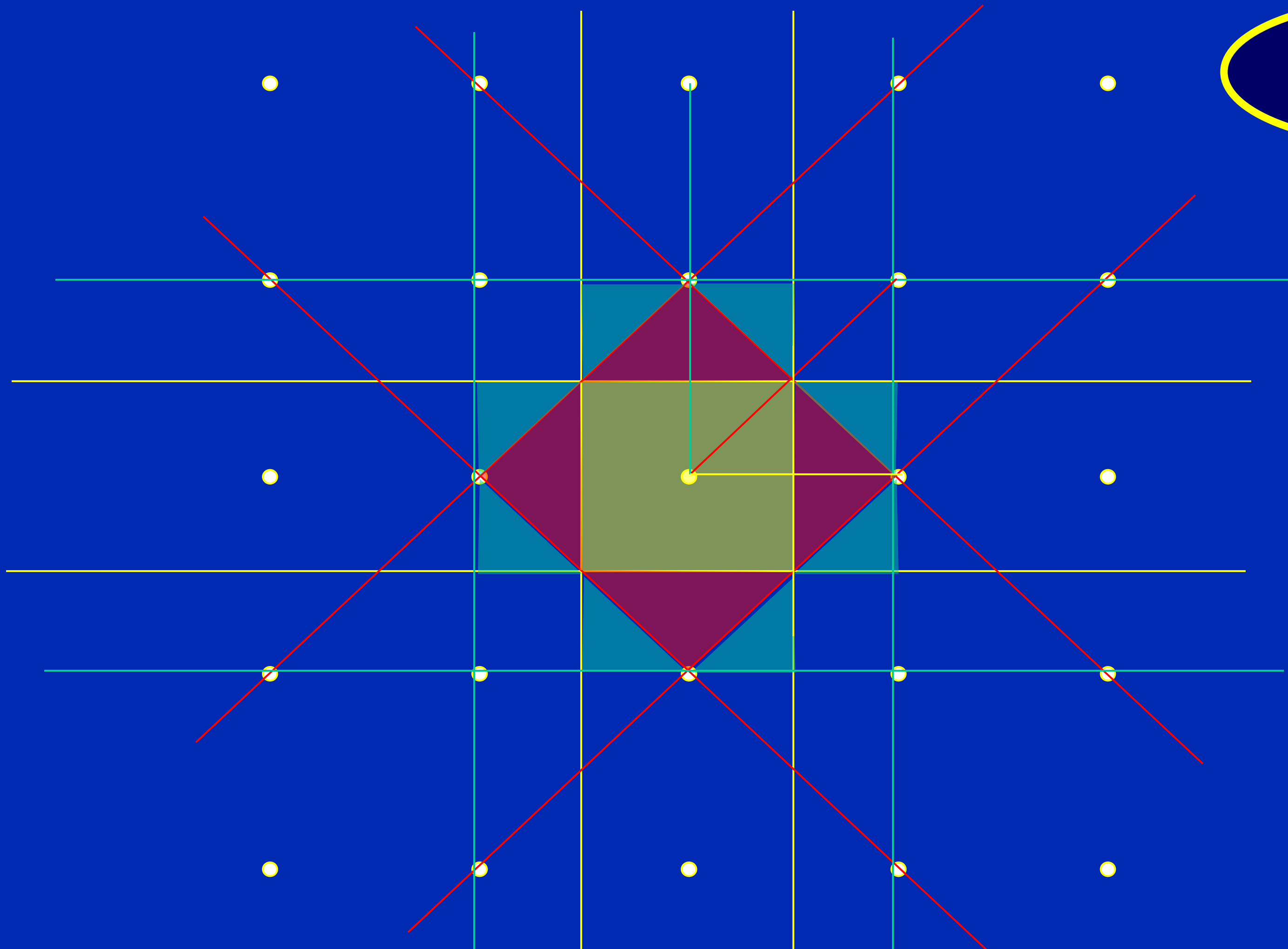


Reduced zone scheme

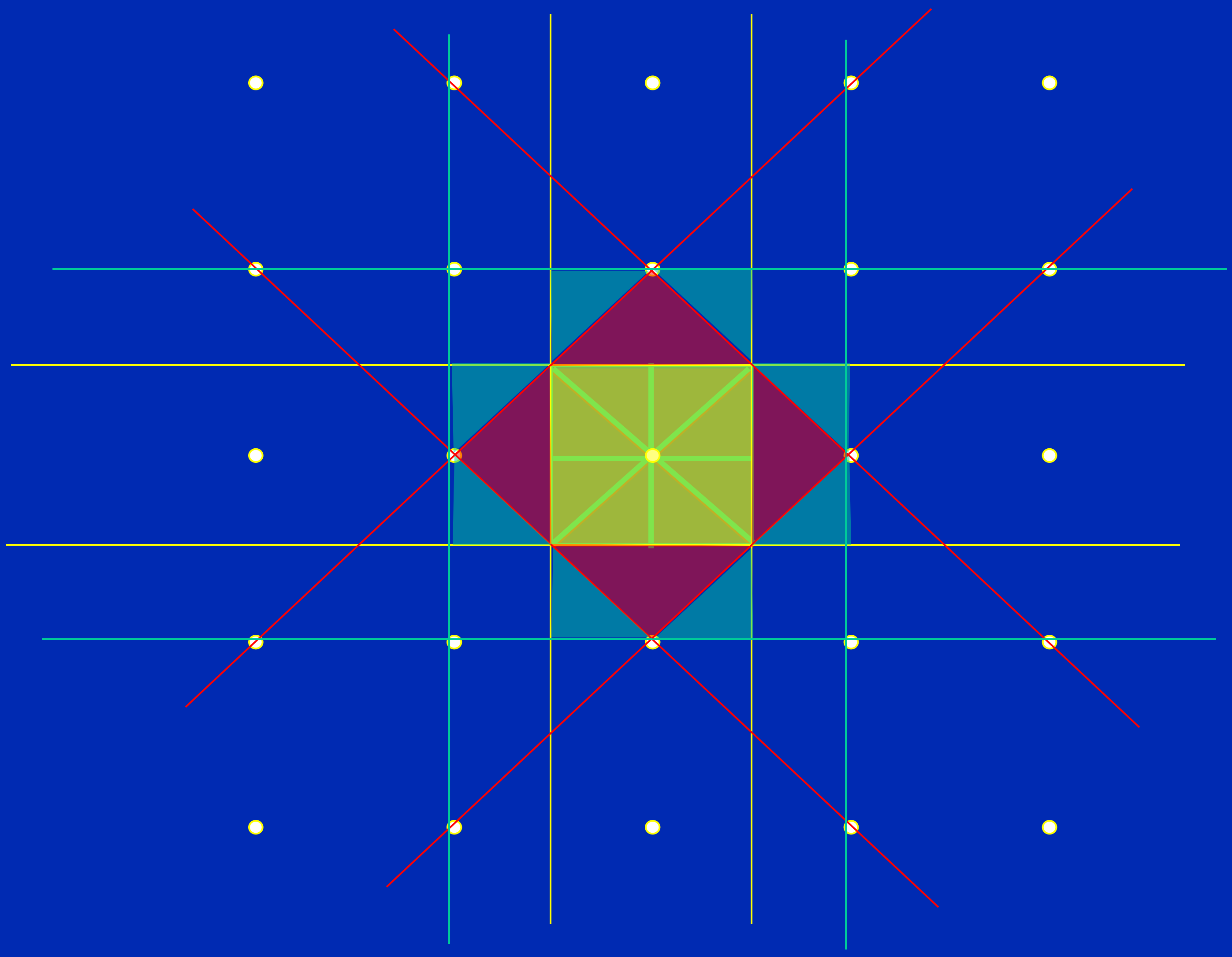


Brillouin zones in 2D

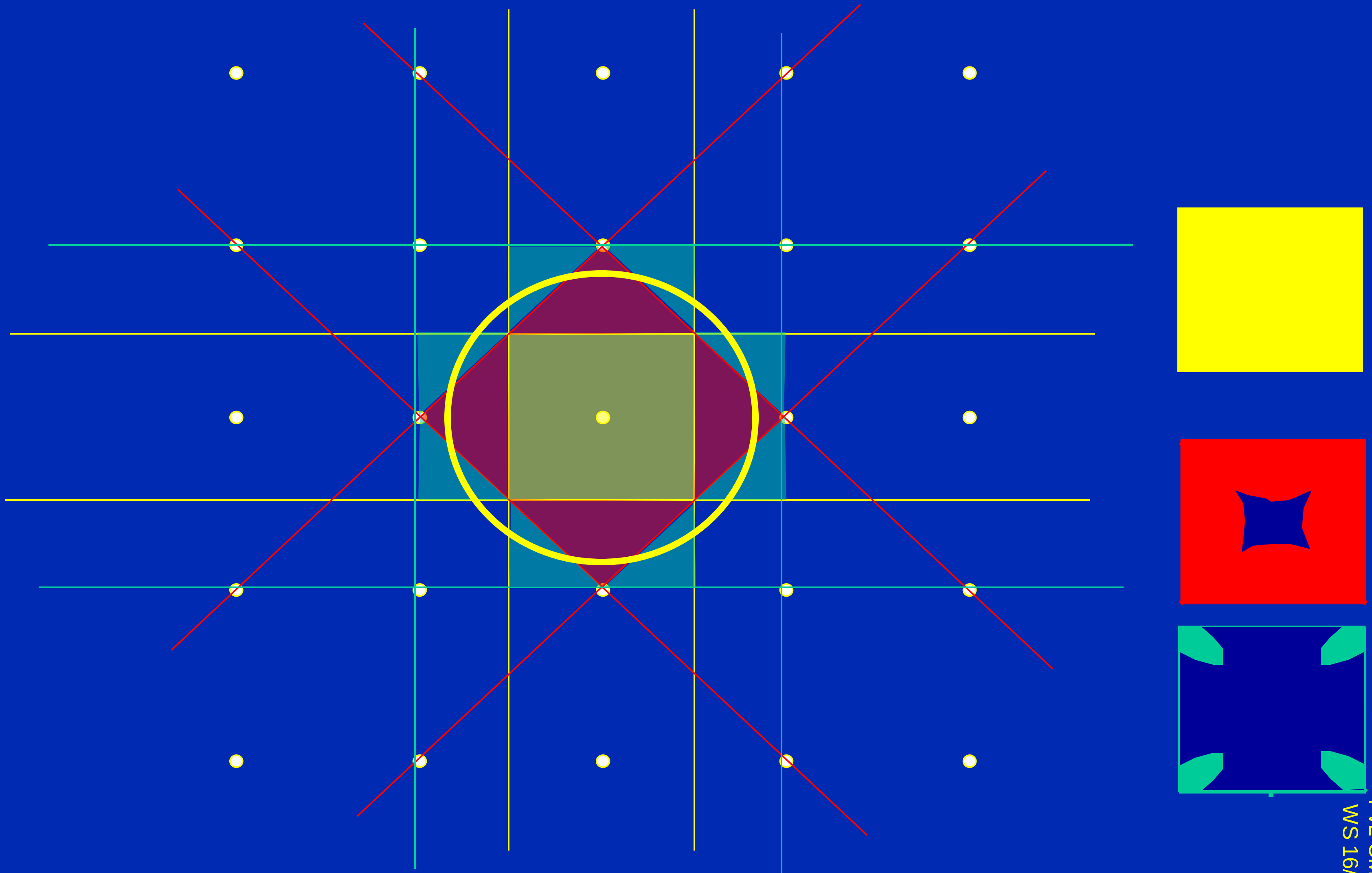
$$\Delta\vec{k} = n \cdot \vec{G}$$



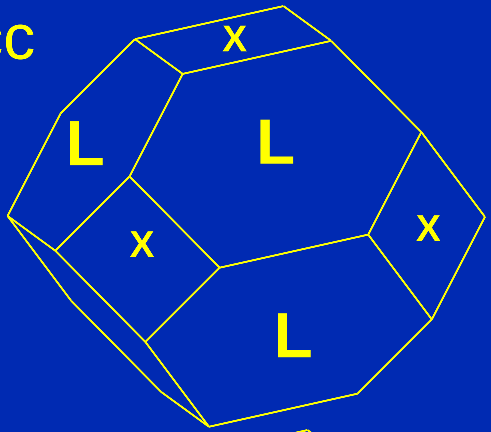
Brillouin zones in 2D



Brillouin zones in 2D



fcc



$N_e = 1$ (examples: Cu,Ag,Au)

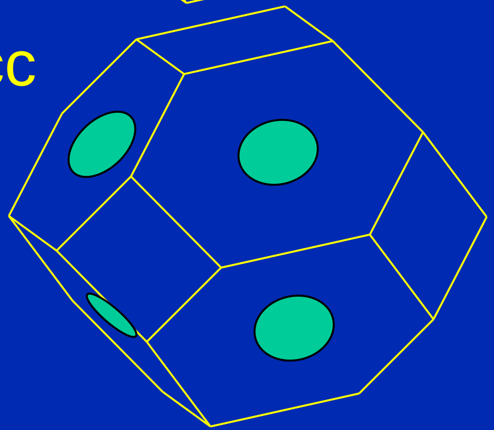
$k_F a/2\pi = (3/2\pi)^{1/3} = 0.78$

$|L|\ a/2\pi = |(0.5,0.5,0.5)| = 0.87$

$|X|\ a/2\pi = |(1,0,0)| = 1.0$

$V_{BZ}/(2\pi)^3 = 4/a^3$

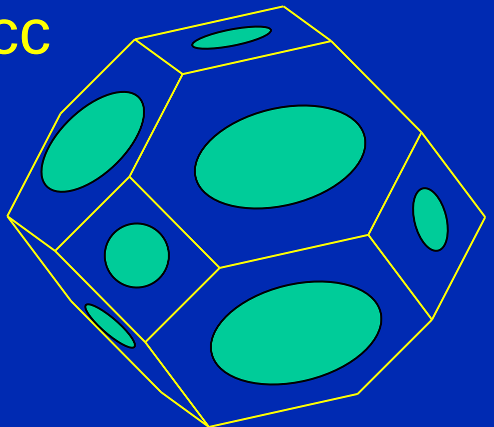
fcc



$N_e = 2$ (examples: Ca, Sr)

$k_F a/2\pi = (3/\pi)^{1/3} = 0.98$

fcc



$N_e = 3$ (examples: Al, Ce, Th)

$k_F a/2\pi = (9/\pi)^{1/3} = 1.13$

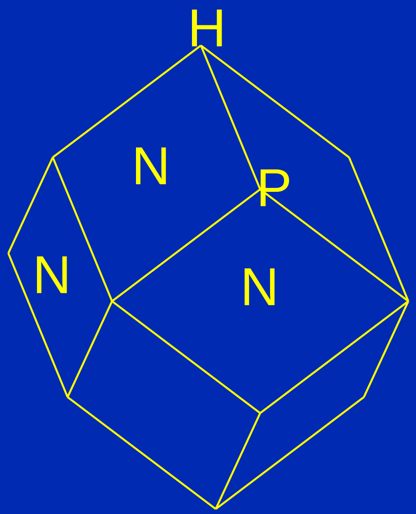
$|N|\, a/2\pi = |(0.5,0.5,0)| = 0.71$

$|P|\, a/2\pi = |(0.5,0.5,0.5)| = 0.87$

$|H|\, a/2\pi = |(1,0,0)| = 1$

$V_{BZ} /(2\pi)^3 = 2/a^3$

bcc

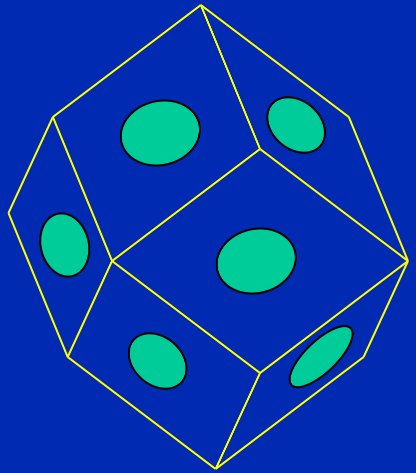


$N_e = 1$

examples: Li,Na,K,Rb,Cs

$k_F a/2\pi = (3/4\pi)^{1/3} = 0.62$

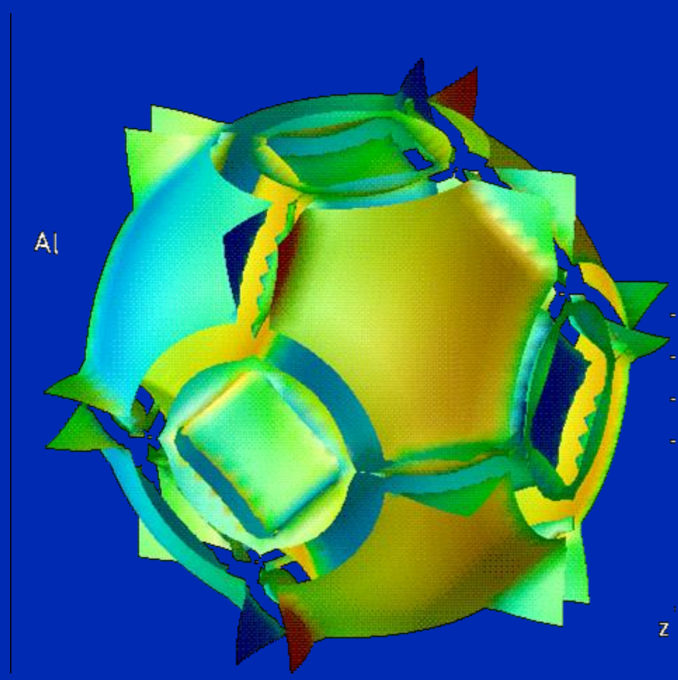
bcc

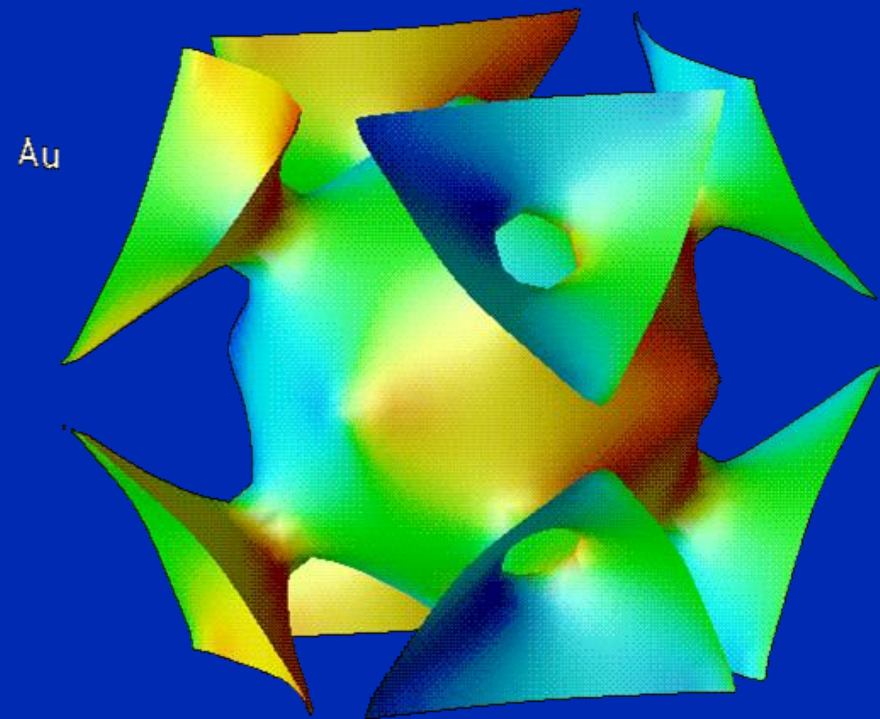


$N_e = 2$

examples: Ba

$k_F a/2\pi = (6/4\pi)^{1/3} = 0.78$


Al
z


Au

Free e$^-$ + periodic potential

- Band structure, gaps, metals & insultators
- Effective mass $$E = \frac{\hbar^2 k^2}{2m^*} \quad \rightarrow \quad m^*(k) = \hbar^2 \left[\frac{\partial^2 E}{\partial k^2} \right]^{-1}$$
- E.O.M. (see Kittel appendix E & pages 205-206)
- Fermi surface
    - Constant E surface for relevant electrons

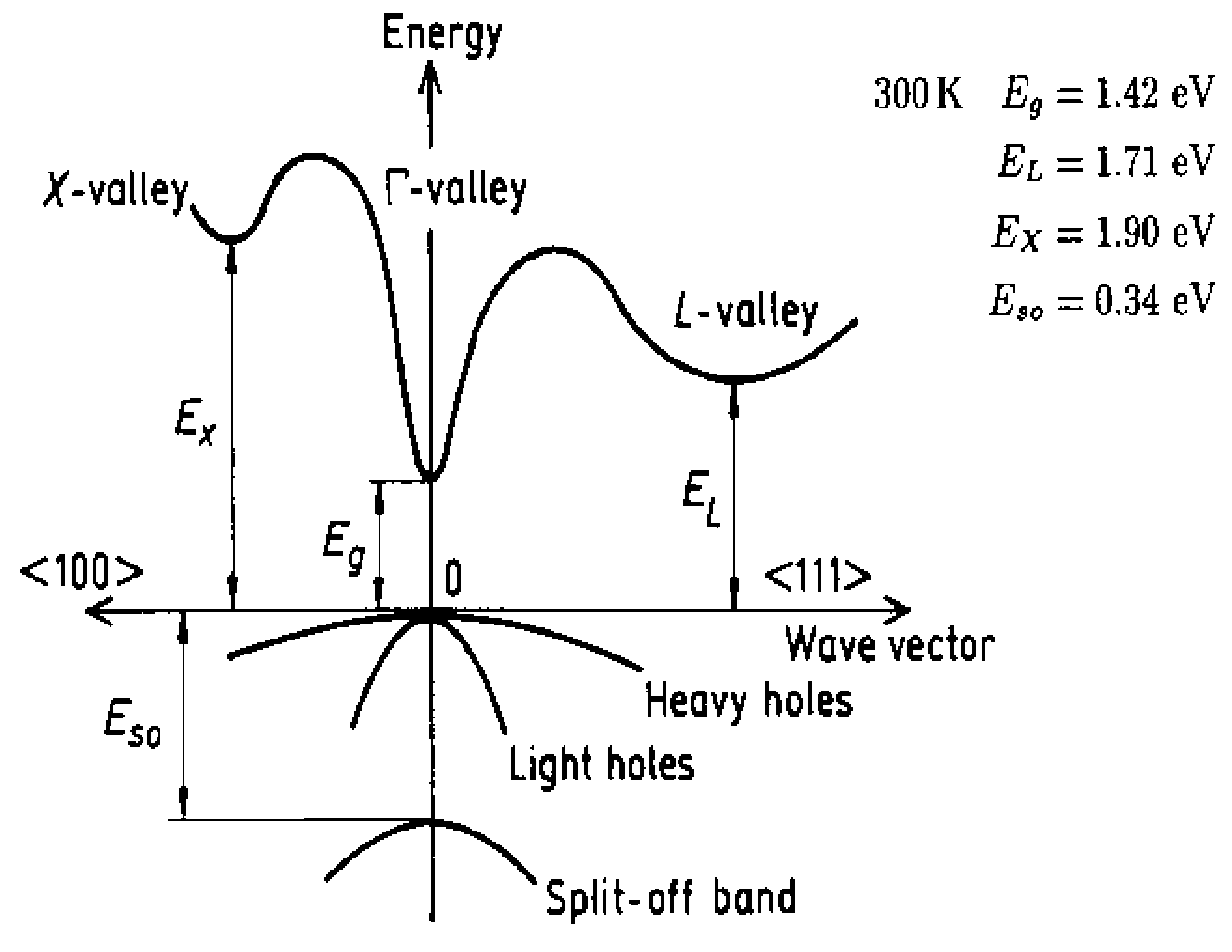
SEMI-CONDUCTORS

Kittel Ch. 8, Ch. 19

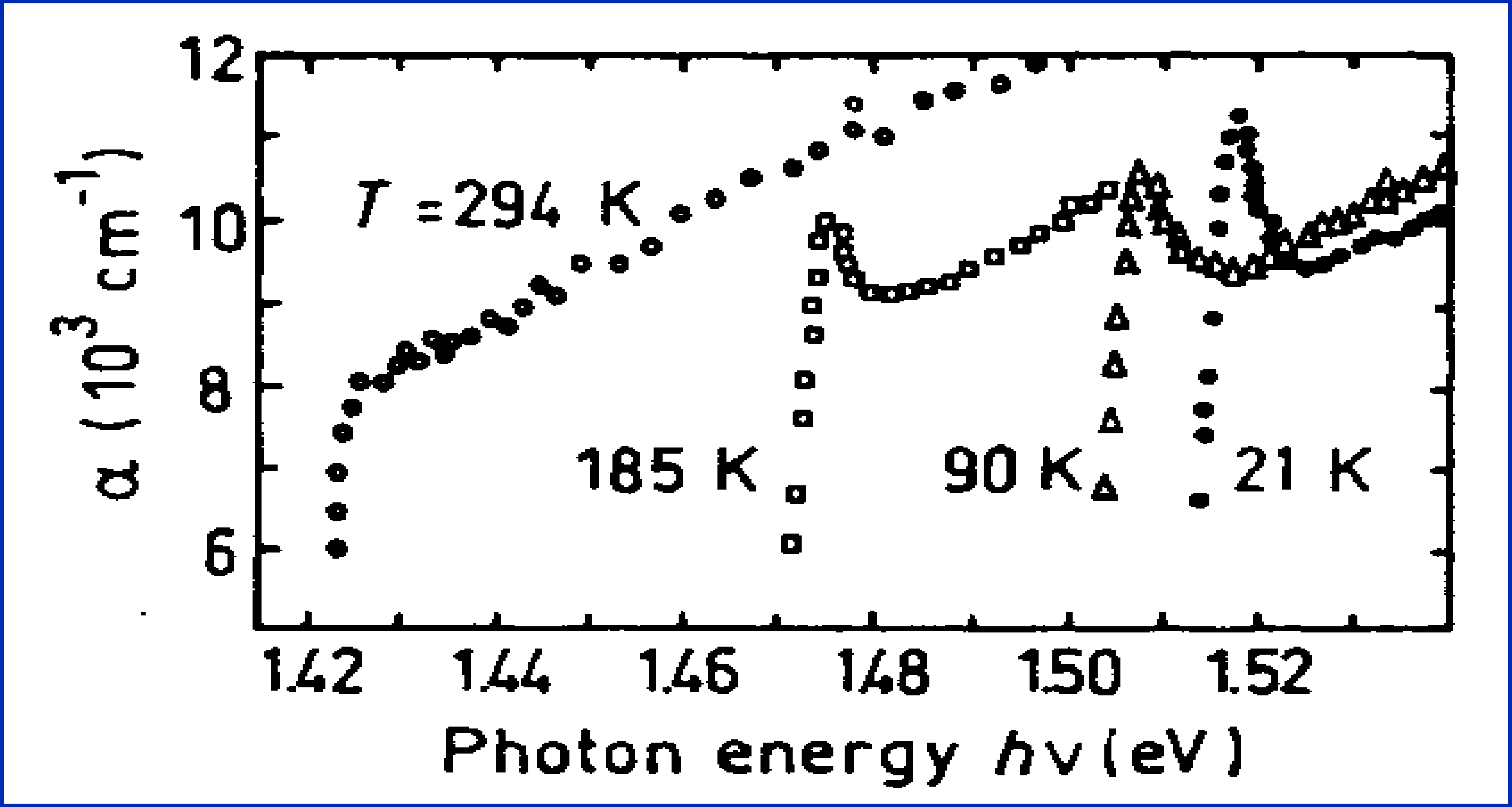
Semiconductors

Direct gap / Indirect gap
Intrinsic / Extrinsic
Homogeneous / Inhomogeneous

Direct gap: GaAs

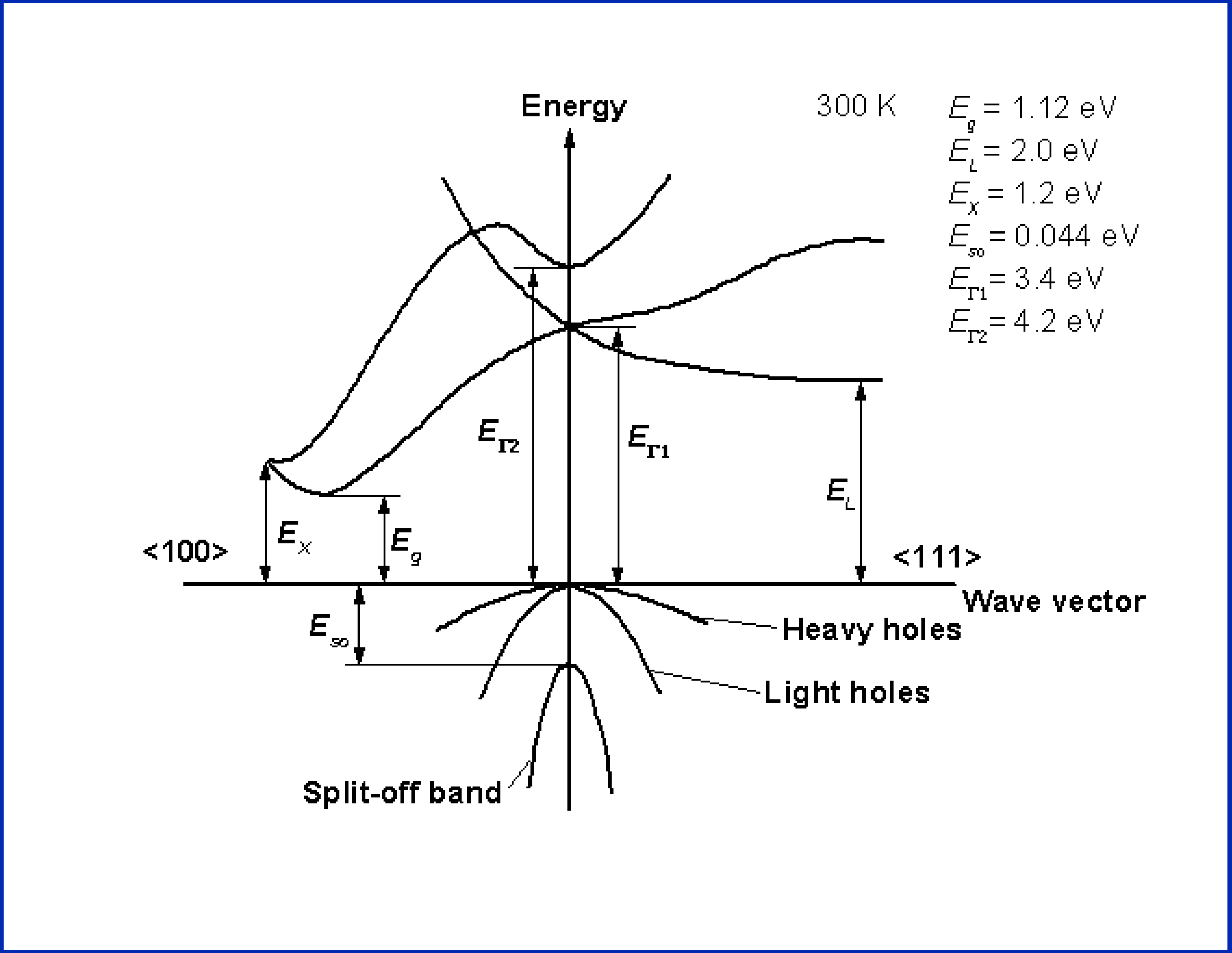


Direct gap: GaAs, absorption

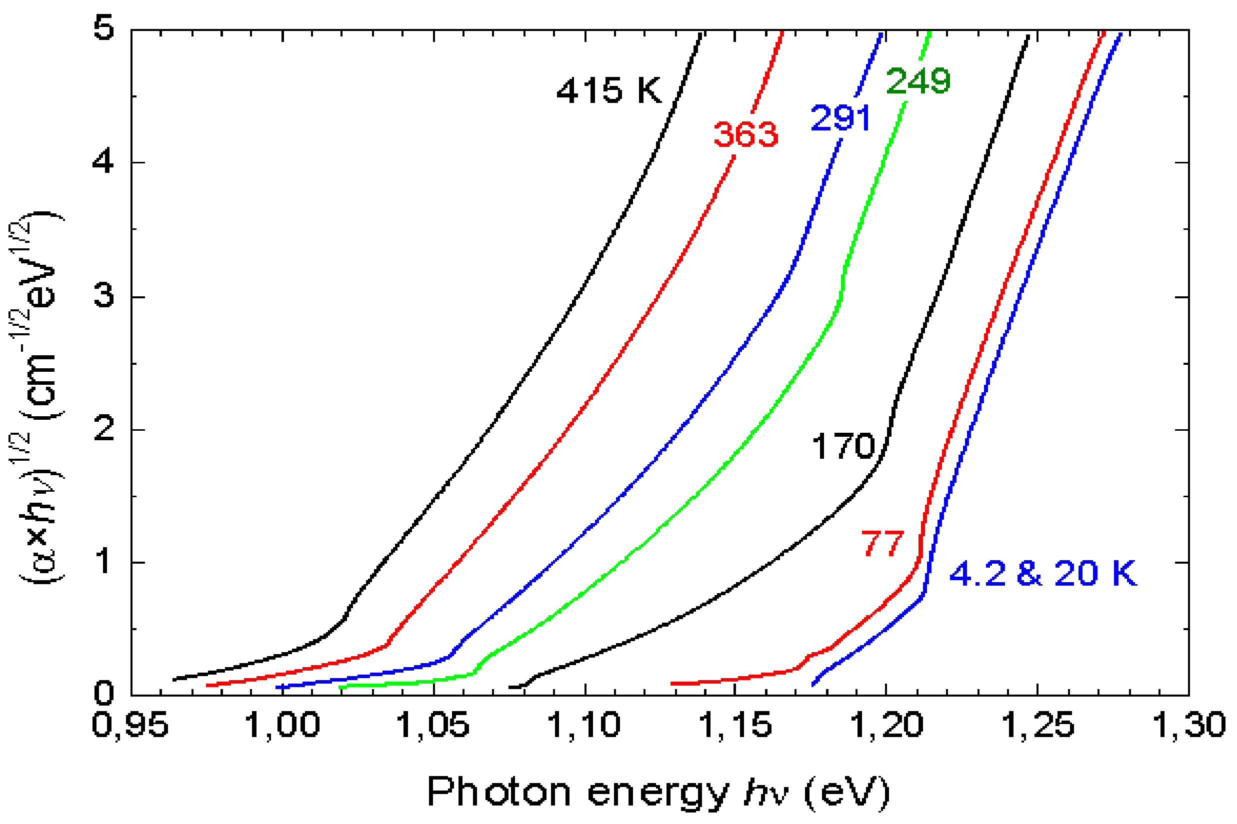


$$I(\omega) = I_0 e^{-\alpha(\omega)\cdot d}$$

Indirect gap: Silicon



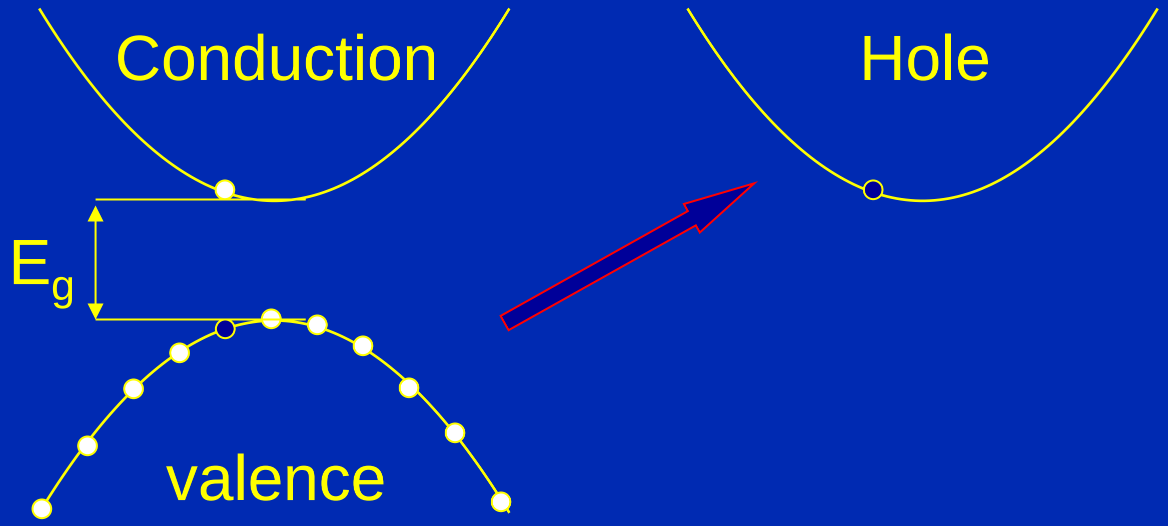
Indirect gap, Si absorption



Holes

Missing electron in a filled band acts as a particle (*hole*) with:

- $k_h = -k_e$
- $E_h = -E_e$
- $v_h = v_e$
- $m_h = -m_e$
- $q_h = -q_e$
- $f_h = 1-f_e$



Cyclotron resonance

$$\omega_c = \frac{eB}{m^*}$$

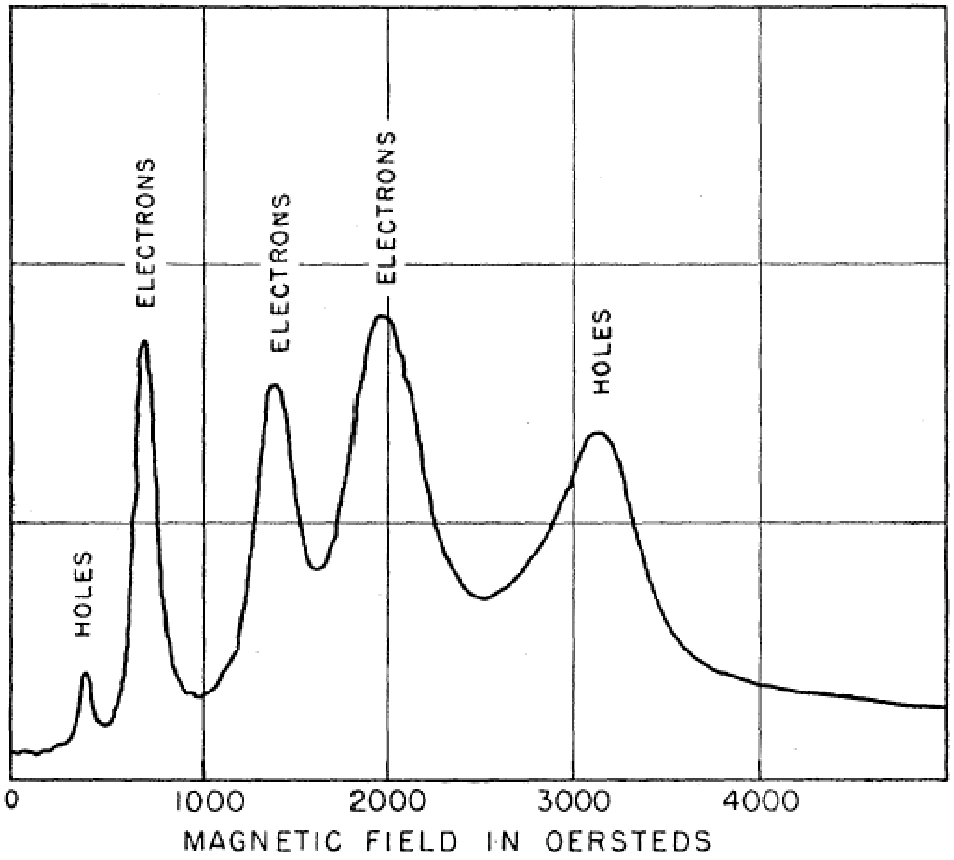
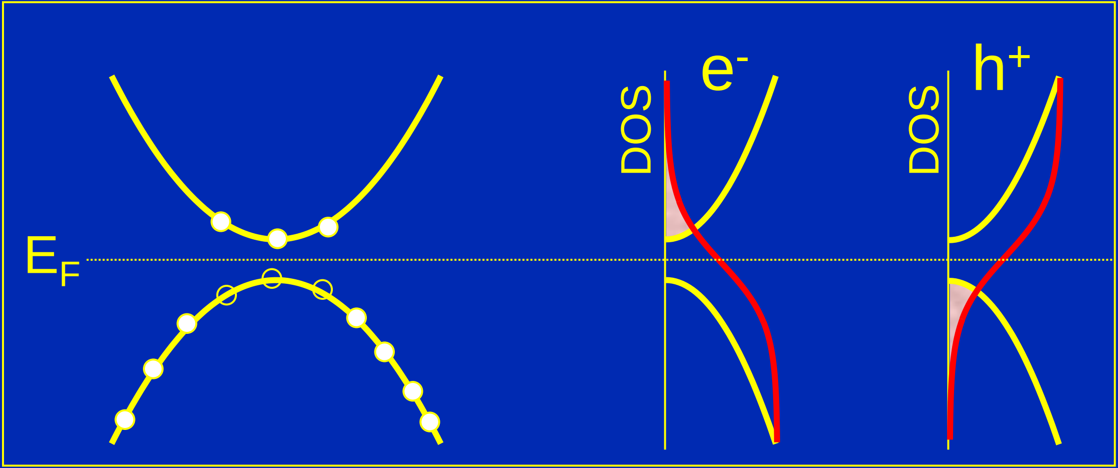


FIG. 2. Typical cyclotron resonance results in germanium near

Dresselhaus et al., Phys. Rev. **98**, 368 (1955)

Carrier density



$$n = \int_{E_c}^{\infty} dE\, D_c(E) \cdot f_e(E) = n_o \cdot e^{\frac{\mu - E_c}{k_b T}} \qquad n_o = 2\left(\frac{m_c^* k_b T}{\pi \hbar^2}\right)^{3/2}$$

$$p = \int_{-\infty}^{E_v} dE\, D_v(E) \cdot f_h(E) = p_o \cdot e^{\frac{E_v - \mu}{k_b T}} \qquad p_o = 2\left(\frac{m_v^* k_b T}{\pi \hbar^2}\right)^{3/2}$$

$$n \cdot p = n_o p_o \cdot e^{-\frac{E_g}{2 k_b T}}$$

Independent of μ or doping

Intrinsic case

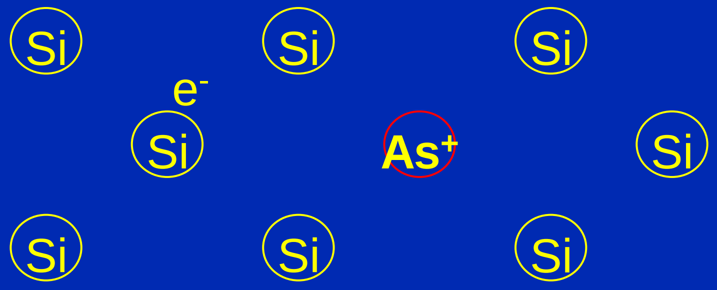
Density: $n_i \equiv p_i = \sqrt{n_o p_o} \cdot e^{-\frac{E_g}{k_b T}}$

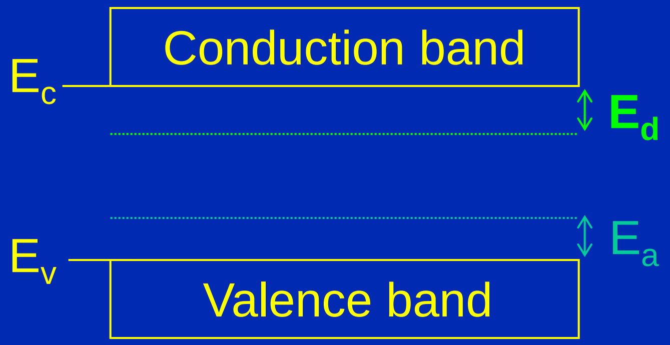
From n = p :

$$n_o \cdot e^{\frac{\mu - E_c}{k_b T}} = p_o \cdot e^{\frac{E_v - \mu}{k_b T}}$$

$$E_F = \mu = \frac{1}{2} E_g + \frac{3}{4} k_b T \cdot \ln\left(\frac{m_h^*}{m_e^*}\right)$$ (setting E_v=0)

Extrinsic case





'H problem' with $e^2 \rightarrow e^2/\varepsilon$ & $m \rightarrow m^*$

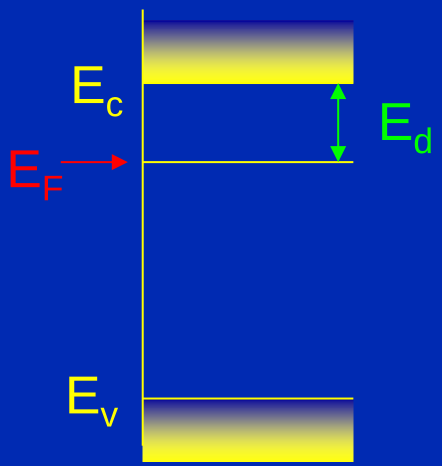
Ionization energy 1 'Ry': $E_d = \frac{m^* e^4}{2\hbar^2 \varepsilon^2} = \frac{m^*}{m_o} \frac{1}{\varepsilon^2} \cdot 13.6\ \text{eV}$

'Bohr' radius: $r_d = \frac{\hbar^2 \varepsilon}{m^* e^2} = \frac{m_o}{m^*} \varepsilon \cdot a_o$

Extrinsic

Donor and acceptor levels (meV)

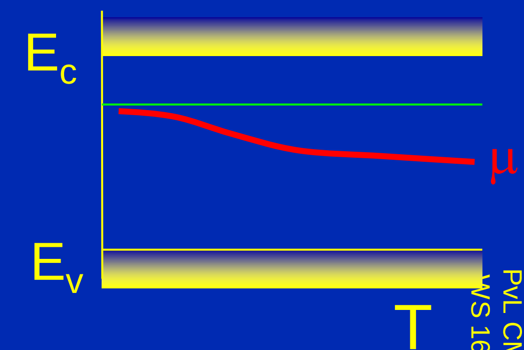
	P	As	Sb	B	Al	Ga	In
Si	45	49	39	45	57	65	157
Ge	12	13	10	10	10	11	11



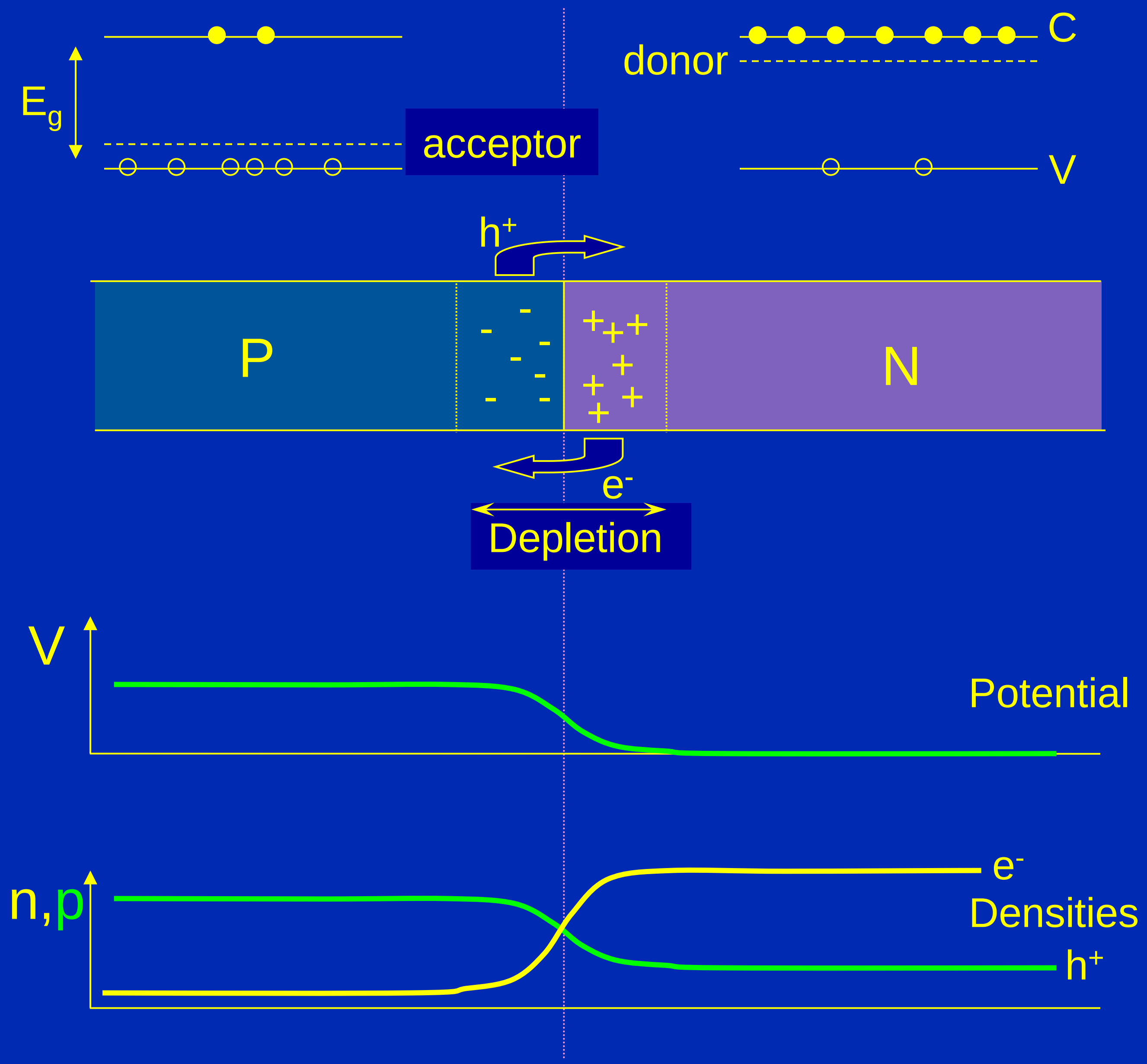
$$N_d^o = N_d \cdot \langle n \rangle = N_d \frac{e^{-(\varepsilon_d-\mu)/kT}+e^{-(\varepsilon_d-\mu)/kT}}{1+e^{-(\varepsilon_d-\mu)/kT}+e^{-(\varepsilon_d-\mu)/kT}} = N_d \frac{1}{\frac{1}{2}e^{(\varepsilon_d-\mu)/kT}+1}$$

$$n_d = N_d - N_d^o = N_d\left(1-\frac{2}{e^{(\varepsilon_d-\mu)/kT}+2}\right)$$

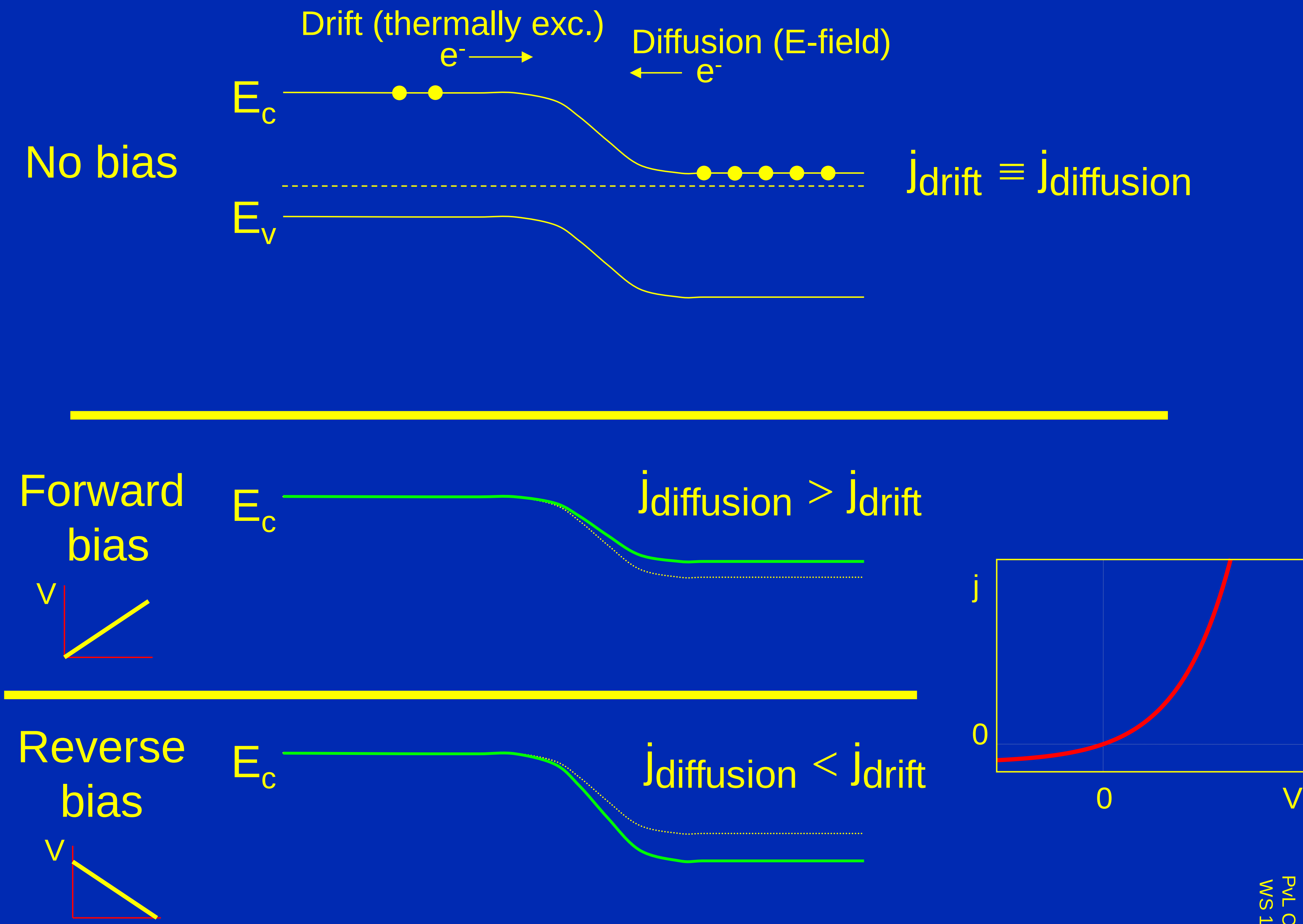
$$\left.\begin{array}{l} n_c = p_i + n_d \\ p_i \approx n_i \end{array}\right\} \rightarrow n \approx \sqrt{N_d n_0} \cdot e^{-E_d/2k_bT}$$



P-N Junction



C
donor
E_g
acceptor
V
h^+
P
N
e^-
Depletion
V
Potential
n,p
e^-
Densities
h^+


Drift (thermally exc.)
e⁻
Diffusion (E-field)
e⁻
E_c
E_v
No bias
$j_{drift} \equiv j_{diffusion}$
Forward bias
E_c
$j_{diffusion} > j_{drift}$
V
Reverse bias
E_c
$j_{diffusion} < j_{drift}$
V
j
0
0
V

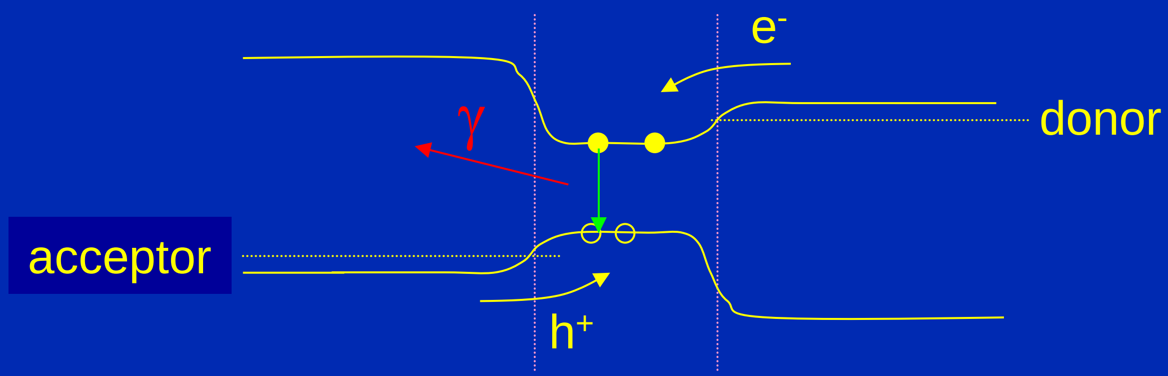
Other heterogeneous S.C.

Recombination: $e^- + h^+ \rightarrow \gamma$

LED and Semiconductor laser

$GaAs_{1-x}P_x$

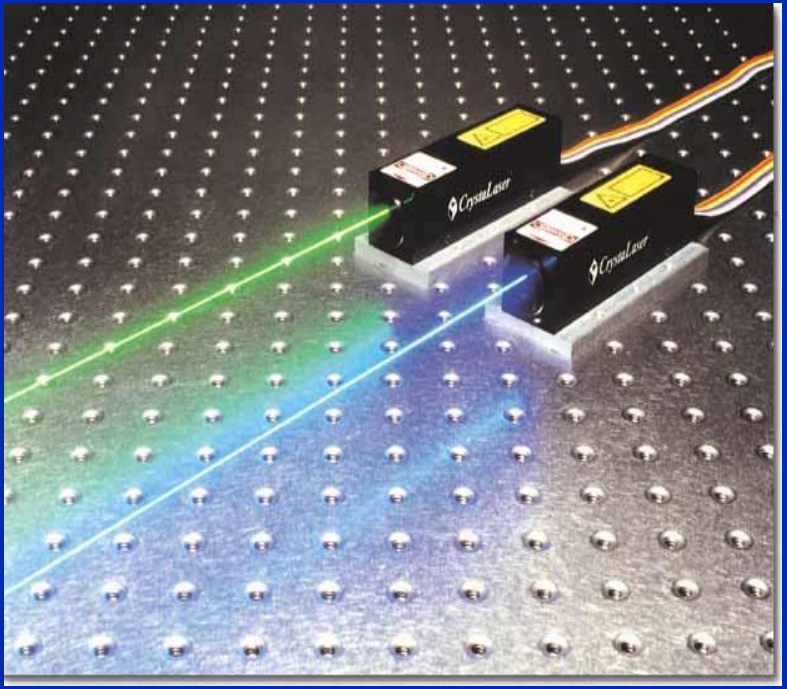
$In_xGa_{1-x}N/Al_yGa_{1-y}N$



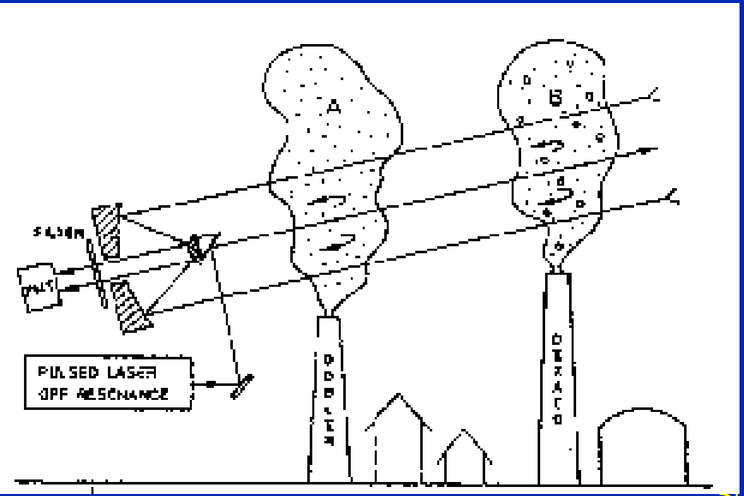
p-type

n-type

SC lasers



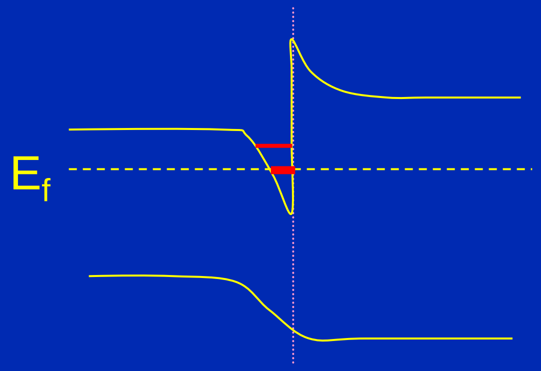
- Storage CD/DVD, MO
- Eye, artery, dental Surgery
- Diagnostic (Caries, Cancer)
- Environmental monitoring
- Remote sensing (speed, chemicals)
- Motion control
- Star Wars, guns
- … …
- …



Heterostructure

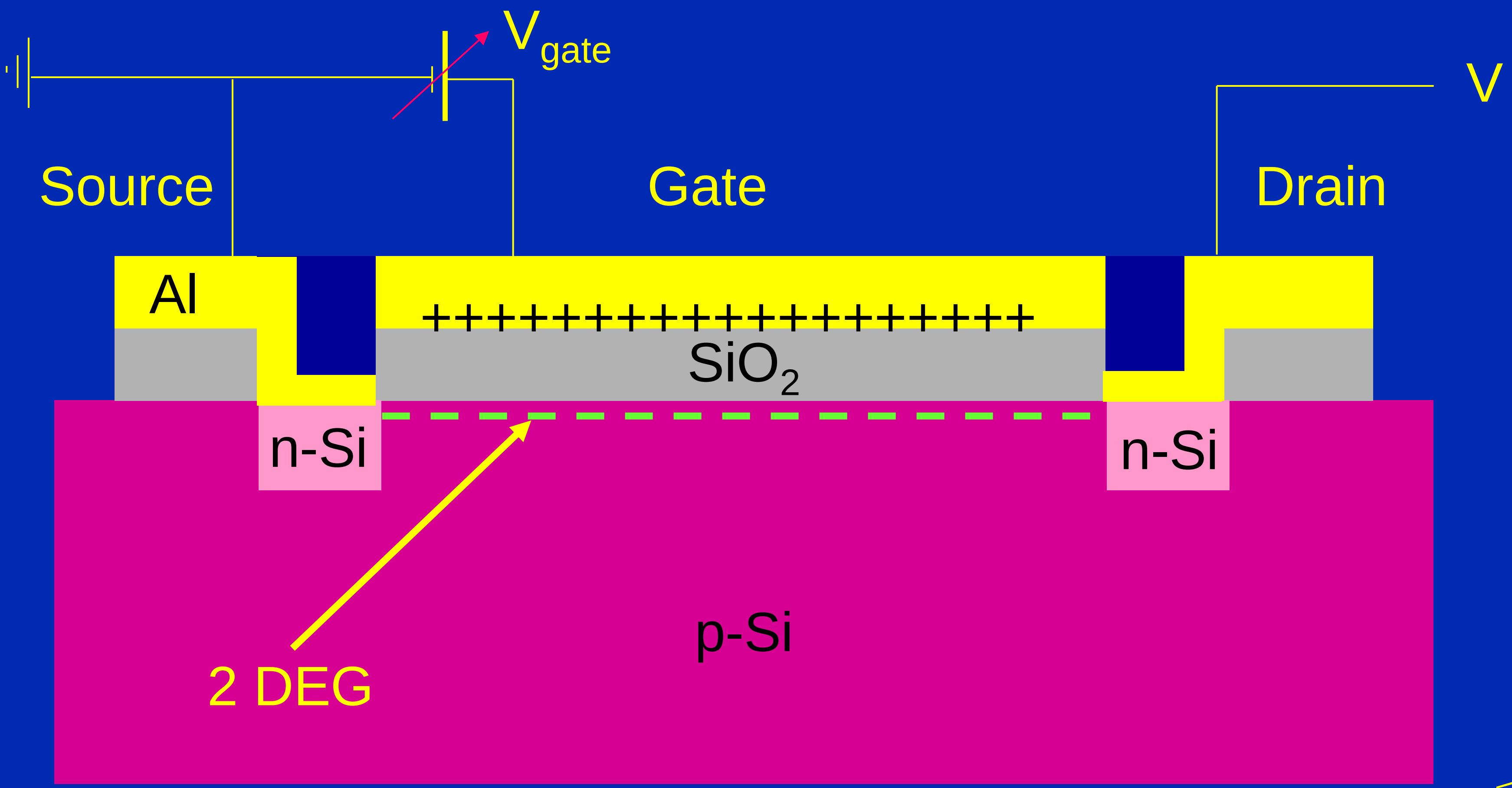
Heterostructure: Lateral confinement => 2 DEG

GaAs

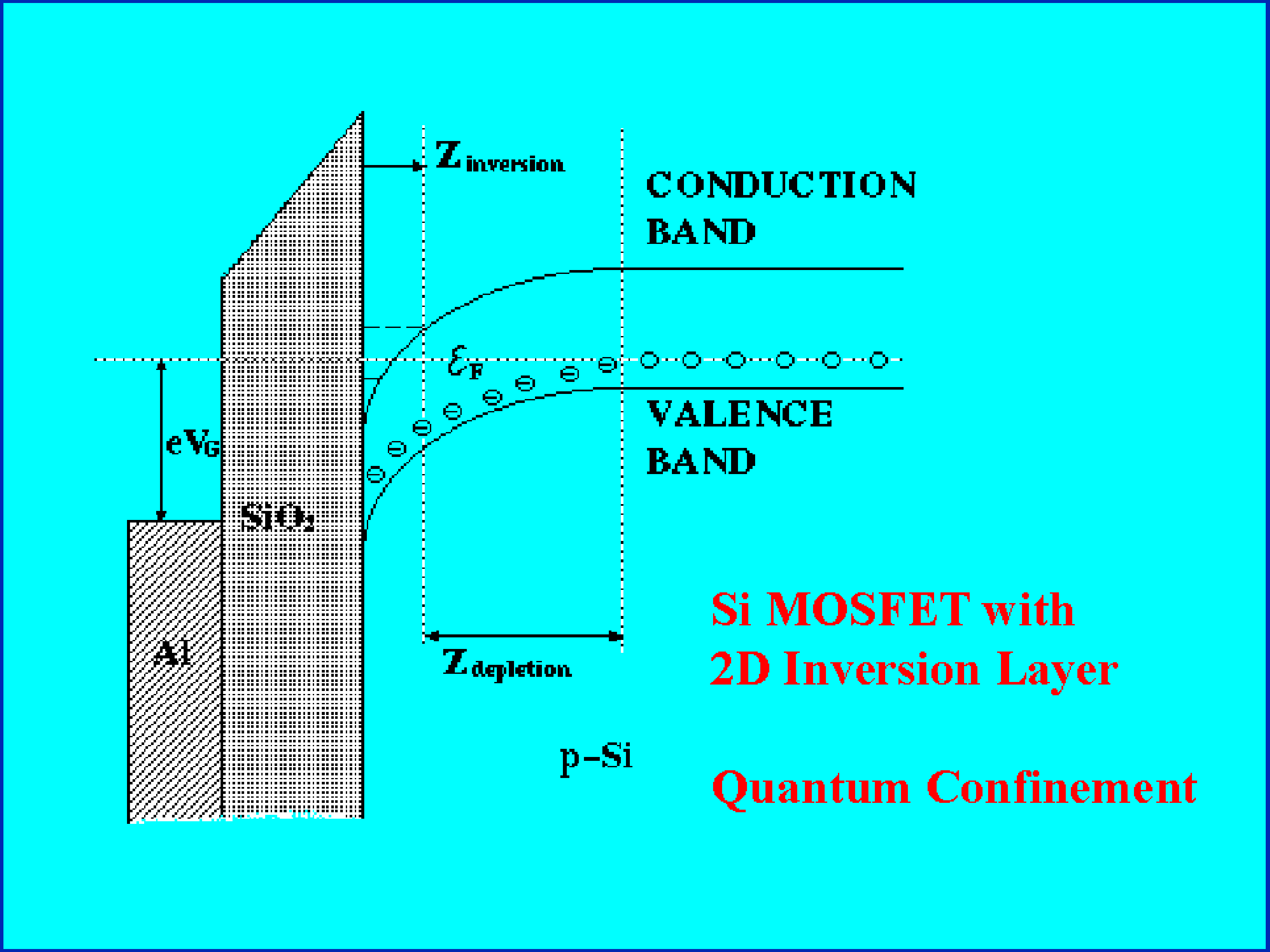


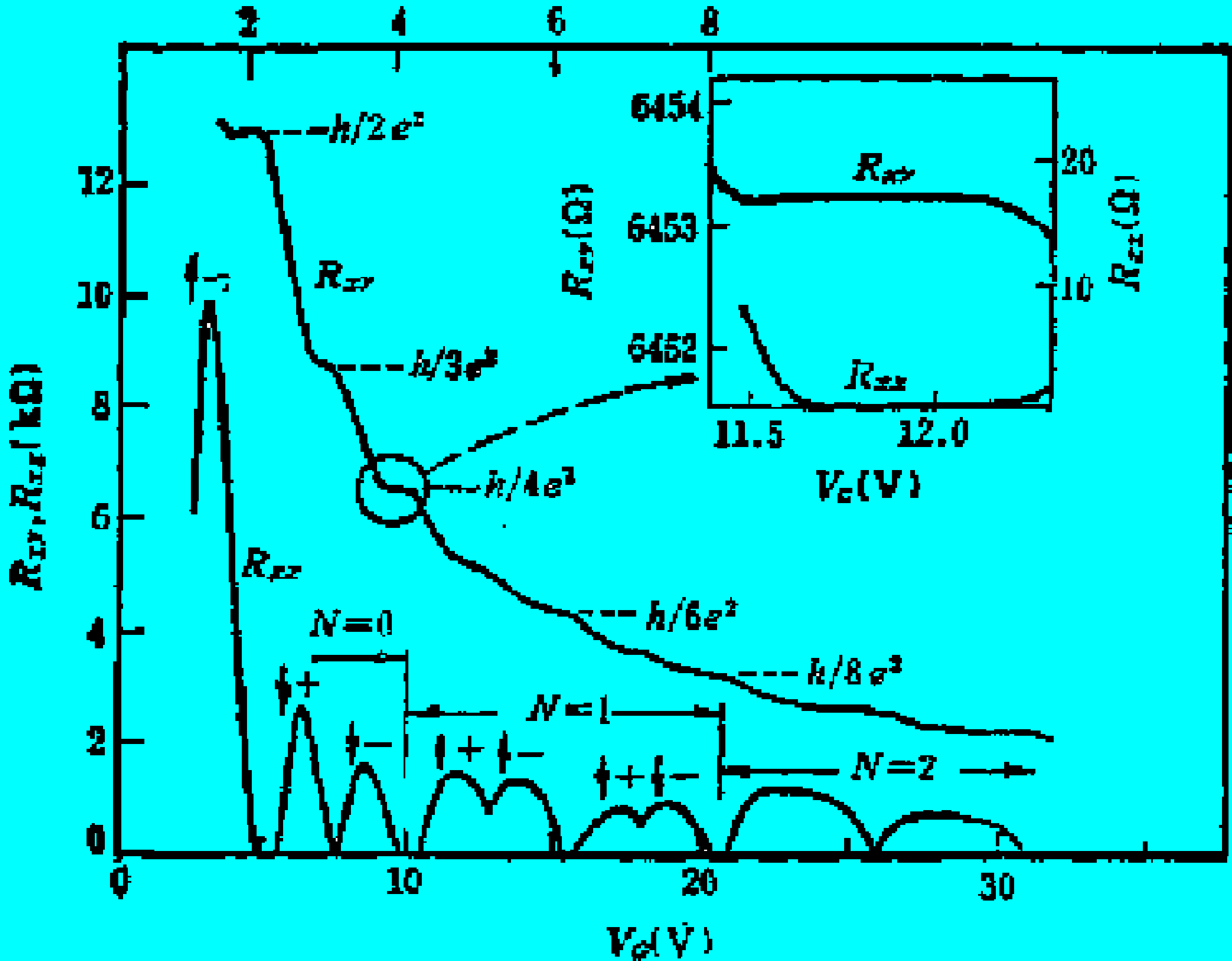
$Al_xGa_{1-x}As$ (n-type)

MOSFET 2DEG



Band diagram MOSFET





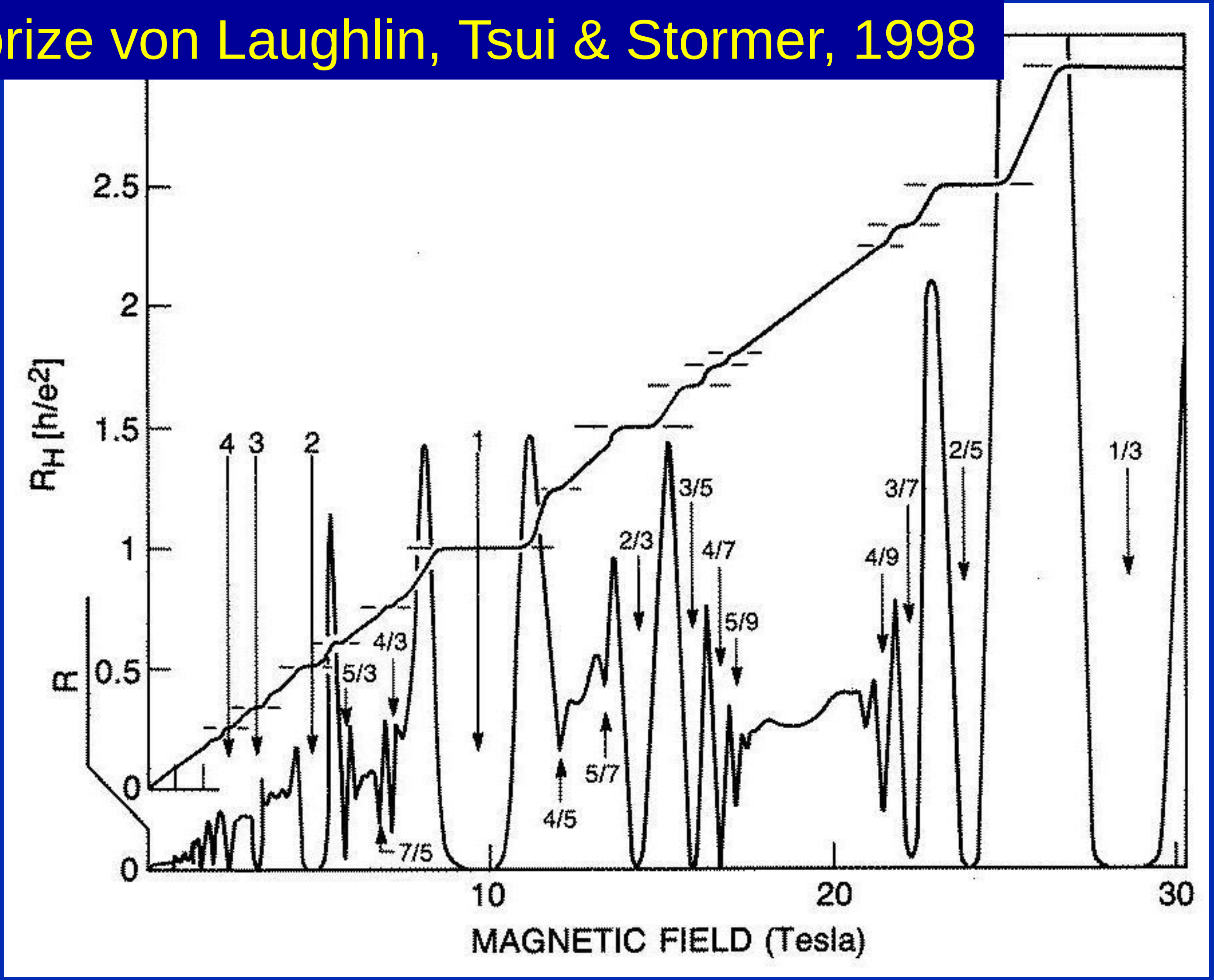
Integer Quantum Hall effect on Si-MOSFET

Nobel prize von Klitzing, 1985

PvL CMP-I

Fractional quantum hall effect

Nobel prize von Laughlin, Tsui & Stormer, 1998



Composite fermions