RKKY (Friedel oscill.)

$$\chi_{pauli} = \frac{3}{2}\,\frac{n\mu_0 \mu_B^2}{2E_F}$$

$$\chi_{landau} = -\frac{1}{3}\chi_{pauli}$$

$$\Bigg\}\quad \chi_{\substack{el\\gas}} = \frac{n\mu_0\mu_B^2}{2E_F}$$

This is response to homogeneous field

what about spatially varying fields?

assume $\bar{H}(\bar{r}) = \bar{H}_{\bar{q}} \cos \bar{q}\cdot\bar{r}$

then perturb. of e^--gas : $\mathcal{H} = \pm \frac{g\mu_0\mu_B}{2} |H_q| \cos \bar{q}\cdot\bar{r}$

this will perturb plane wave states e^- gas: $\psi_{k\pm}(r) = \frac{1}{\sqrt{V}} e^{ikr} |\pm\rangle$ (spin)

in particular the couple to wfncts $e^{i\bar{k+}\bar{r}}$ & $e^{i\bar{k-}\bar{r}}$

2^{nd} order pert. calc. : $|\psi\rangle = \sum_j a_j |\varphi_j\rangle$

with $a_j = \frac{V_{ik}}{E_k - E_i}$

$$\Rightarrow \psi_{k\pm}(r) = \frac{1}{\sqrt{V}} \cdot \left\{ e^{ikr} \pm \frac{g\mu_0\mu_B H_q}{4} \left[\frac{e^{i(\bar{k}+\bar{q})r}}{E_{k+q} - E_k} + \frac{e^{-i(\bar{k}-\bar{q})r}}{E_{k-q} - E_k} \right] \right\} \cdot |\pm\rangle$$

with $E_k = \frac{\hbar^2 k^2}{2m}$

Leading order H_q $\Rightarrow$ $|\psi_{k\pm}(r)|^2 = \frac{1}{V}\left\{1 \pm \frac{g\mu_0\mu_B^2 H_q m_e}{\hbar^2}\left[\frac{1}{(\bar{k}+\bar{q})^2-k^2} + \frac{1}{(\bar{k}-\bar{q})^2-k^2}\right]\cos\bar{q}\cdot\bar{r}\right\}$

magnetization of this wave function:

$$M(r) = \frac{g\mu_0\mu_B}{2}\left(|\psi_{k+}(r)|^2 - |\psi_{k-}(r)|^2\right)$$

$$= \frac{g^2\mu_0\mu_B^2 m_e H_q \cos\bar{q}\cdot\bar{r}}{\hbar^2 V}\cdot\left[\frac{1}{(\bar{k}+\bar{q})^2-k^2} + \frac{1}{(\bar{k}-\bar{q})^2-k^2}\right]$$

For total $\bar{e}$-gas (at $T=0$) integrate over states:

$$M(r) = M_q \cos\bar{q}\cdot\bar{r}$$

$D(k) = V\frac{k^2}{\pi^2}$

$$\text{with } M_q = \frac{g^2\mu_0\mu_B^2 m_e H_q}{\hbar^2 V}\int_{|\bar{k}|<k_F} D(k)\cdot d^3k\cdot\left[\frac{1}{(\bar{k}+\bar{q})^2-k^2} - \frac{1}{(\bar{k}-\bar{q})^2-k^2}\right]$$

$$= \frac{k_F m_e g^2\mu_0\mu_B^2 H_q}{\pi^2\hbar^2}\left[1 + \frac{4k_F^2-q^2}{4k_F q}\cdot\ln\left|\frac{q+2k_F}{q-2k_F}\right|\right]$$

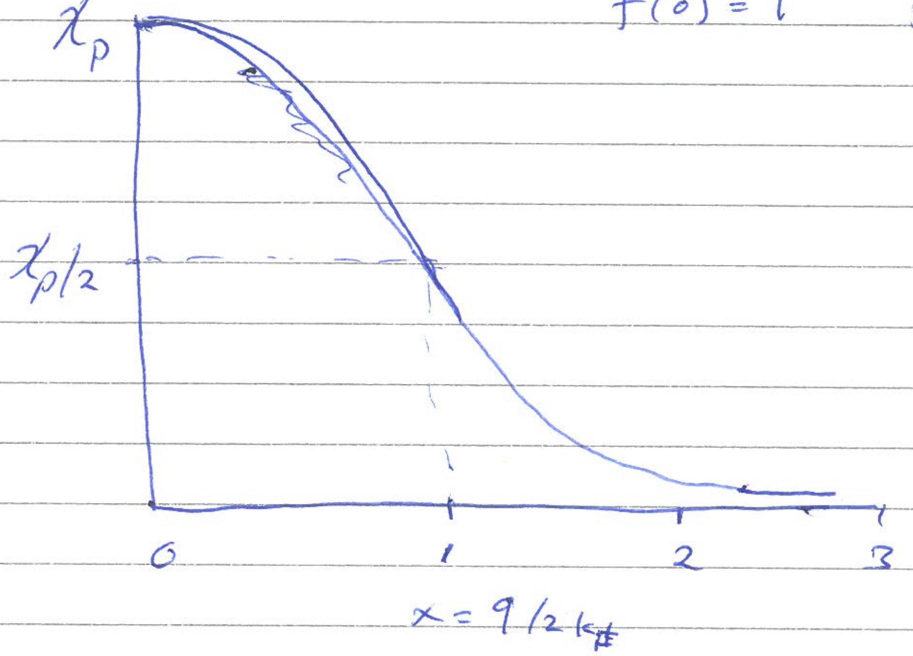
Using $D(E_F) = \frac{m_e k_F}{(\pi\hbar)^2} = \frac{3}{2}\frac{n}{E_F}$, $\chi_p = \frac{3}{2}\frac{n\mu_0\mu_B^2}{2E_F}$

and $g=2$.

$$M = \frac{3}{2}\frac{n\mu_0\mu_B^2}{2E_F}\cdot f\left(\frac{q}{2k_F}\right) = \chi_p\cdot f\left(\frac{q}{2k_F}\right)$$

with $f\left(\frac{q}{2k_F}\right) = f(x) = \frac{1}{2}\left(1 + \frac{1-x^2}{2x} \ln\left|\frac{x+1}{x-1}\right|\right)$.

$f(0) = 1 \qquad f(1) = \frac{1}{2}$.



So response to $H_q \cos q \cdot r$ is know.

more general: $H(r) = \frac{1}{(2\pi)^3} \int d^3q \, H_q \, e^{iqr}$

with $H_q = \int d^3r \, H(r) \, e^{-iqr}$

same for $M(r)$ vs M_q
and $\chi(r)$ vs χ_q.

where $M(r) = \int d^3r' \, \chi(r-r') \, H(r')$

and (convolution theorem) : $M_q = \chi_q \, H_q$

what if we have magnetic impurity $H(r) = \delta(r) H$.

$$H_1(r) = \delta(r) H = \frac{1}{(2\pi)^3}\int H e^{iqr} d^3q.$$

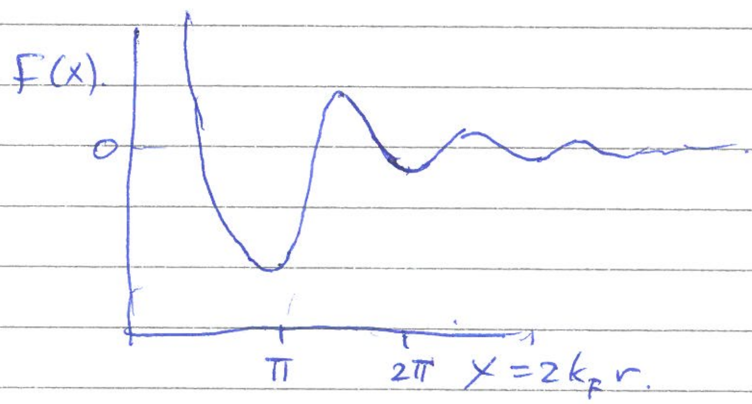
response to single spatial frequency q: χ_q.

then $$\chi(r) = \frac{1}{(2\pi)^3}\int d^3q\, \chi_q e^{iqr}.$$

$$= \frac{1}{(2\pi)^3}\int d^3q \frac{\chi_p}{2}\left(1 + \frac{4k_F^2 - q^2}{4k_F q}\cdot \ln\left|\frac{q+2k_F}{q-2k_F}\right|\right) e^{iqr}.$$

$$= \frac{2k_F^3 \chi_p}{\pi}\cdot F(2k_F r)$$

with $$F(x) = \frac{-x\cos x + \sin x}{x^4}$$



x large (long distance):

$$\chi \sim \frac{\cos(2k_F r)}{r^3}$$ and so is magnetization.