Condensed Matter Physics I

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Previously

Thermal expansion

- Lattice anharmonicity

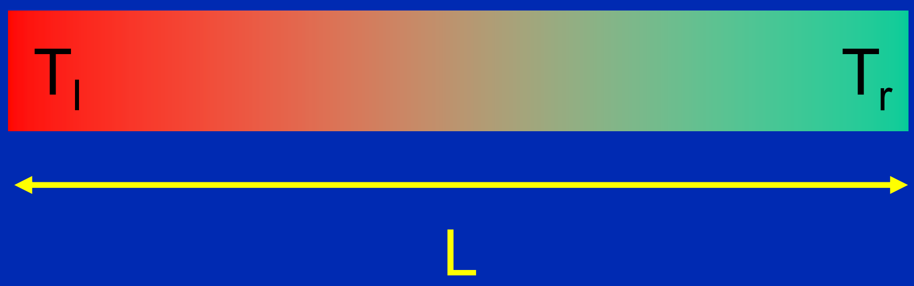
Heat Capacity

- Dulong-Petit law
- Density of states
- Debye model (T^3 law)
- Einstein model
- Phase transitions

Today

- Heat conductivity
- Intro to second quantization
- Metals

Thermal conductivity



Thermal energy flux: $J_u = \frac{dw}{dt}$

Thermal gradient: $\frac{dT}{dx} = \frac{T_r - T_l}{L}$

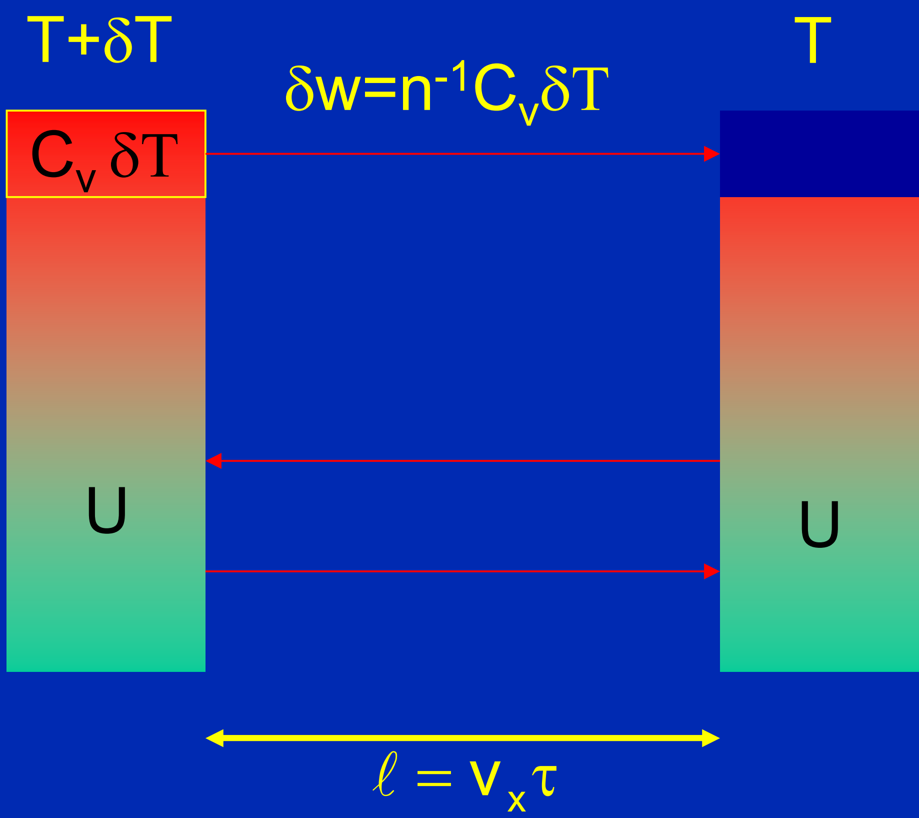
$$J_u = -\kappa \frac{dT}{dx}$$

κ: Thermal conductivity

Is due to diffusion of particles (here phonons)

Thermal conductivity

$$J_u = -\kappa \frac{dT}{dx}$$



$$J_u = \frac{dw}{dt} = n\langle v_x \cdot \delta w\rangle = C_v \langle v_x \delta T\rangle$$

$$\delta T = -v_x \tau \cdot \frac{dT}{dx}$$

$$J_u = -C_v \tau \frac{dT}{dx}\langle v_x^{\,2}\rangle$$

$$\left.\begin{aligned}\langle v_x^{\,2}\rangle &= \frac{1}{3}v^2 \\ \tau &= \ell / v\end{aligned}\right\} \Rightarrow J_u = -\frac{1}{3}C_v v \ell \frac{dT}{dx}$$

$$\kappa = \frac{1}{3}C_v v \ell$$

Scattering of phonons

Phonon mean free path ℓ limited by

- Imperfections, crystal boundaries, isotopes
- Phonon-phonon scattering (normal processes)
- Phonon-phonon scattering (Umklapp processes)

Harmonic approximation in perfect lattice:

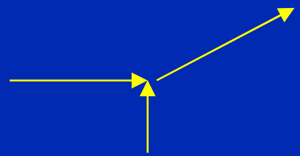
Phonons are eigenstates of Hamiltonian

NO phonon-phonon scattering

Scattering of phonons

Imperfections etc.: ℓ = T independent: $\kappa \propto v \ell C_v(T) \propto T^3$

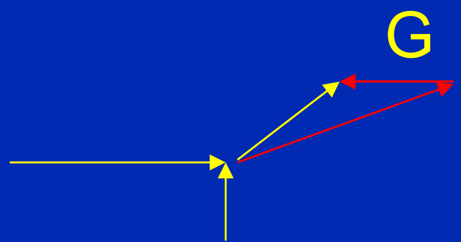
Normal processes: $\hbar\omega_{k_1} + \hbar\omega_{k2} = \hbar\omega_{k3}$



$$\vec{k}_1 + \vec{k}_2 = \vec{k}_3$$

No net change of phonon momentum

Umklapp processes: $\hbar\omega_{k_1} + \hbar\omega_{k2} = \hbar\omega_{k3}$



$$\vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{G}$$

Momentum transferred to crystal: $\hbar\vec{G}$

Thermal conductivity

$$\kappa = \frac{1}{3} C_v v \ell$$

High T: C_v constant, scattering $\propto n(\omega) \propto T$ ⟶ $\kappa \propto T^{-1}$

Low T: Umklapp needs phonon at BZB (~Debye energy)

Very low T: no phonons for U-process:

ℓ constant (imperfections) ⟶ $\kappa \propto T^3$

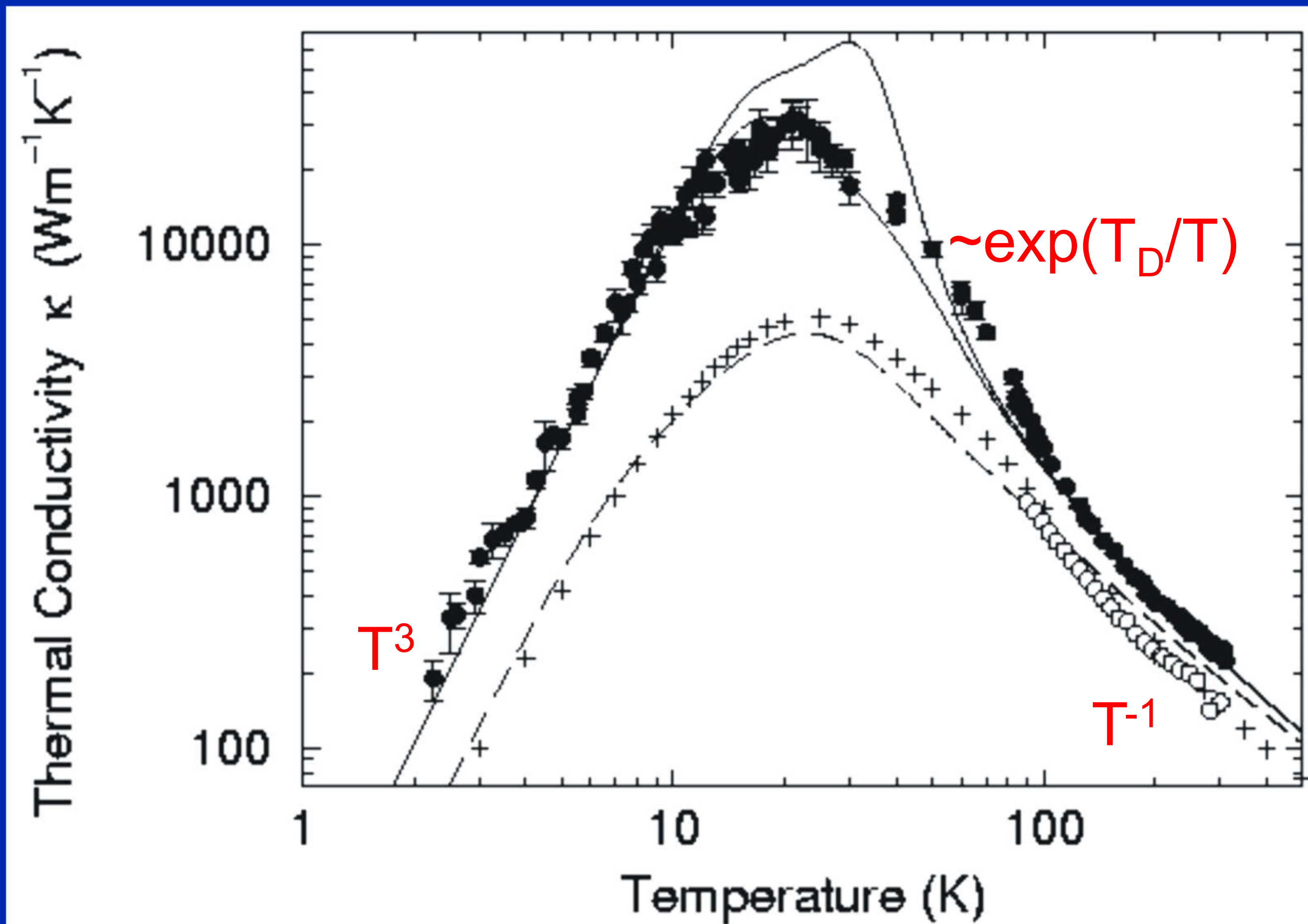
Near T~T_D U-processes become important

Scattering ~ # phonons with ~Debye energy

$$\ell^{-1} \propto n(\hbar\omega_D / 2) = \frac{1}{e^{\theta_D / 2T} - 1} \approx e^{-\theta_D / 2T}$$

$$\kappa \propto e^{\theta_D / 2T}$$

Thermal conductivity



Isotope pure Silicon (T. Ruf et al. Sol. St. Comm. 2000)