# Condensed Matter Physics I

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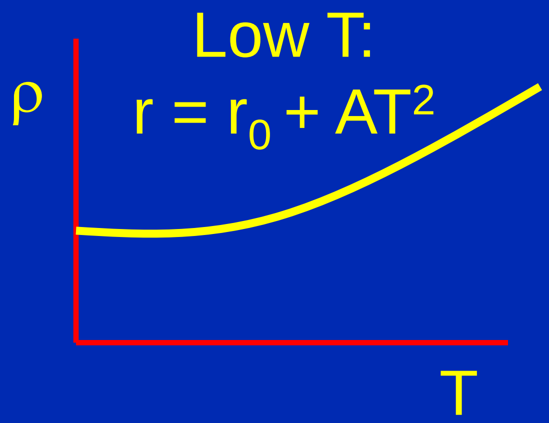
# Metals

# What is a metal ?

Electrical conductivity:

$\rho_{300\,K}$ ~ 1.7 (Cu) - 153 $\mu\Omega\cdot$cm (Pu)




Thermal conductivity:

Cu: $K_{300\,K}$ ~ 3.9 W/Kcm     Pu: $K_{300\,K}$ ~ 0.049 W/Kcm

Wiedemann-Franz: $K/\sigma = \alpha\, T$

Quartz: K ~ 0.13 W/Kcm     NaCl: K ~ 0.27 W/Kcm

Reflectivity:

Highly reflecting upto plasma-frequency

$\omega < \omega_p \qquad \omega_p^2 = 4\,\pi\, n\, e^2 / m$

# FREE ELECTRON MODEL

# FEM, overview

- Free electron model (Drude, Sommerfeld theory)
- Statistics and density of states
- Heat capacity
- Electrical conductivity (Ohm's law)
- Influence of a magnetic field (Hall effect)
- Thermal conductivity and Wiedemann-Franz law

# Electrons in metals

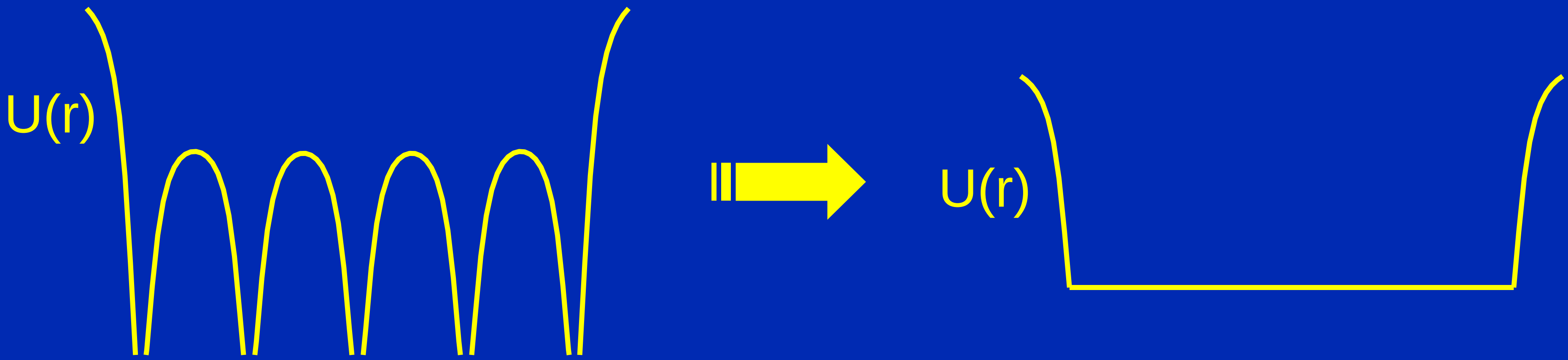
P. Drude: 1900 kinetic gas theory of electrons, classical
Maxwell-Boltzmann distribution
independent electrons
free electrons
scattering from ion cores (relaxation time approx.)

A. Sommerfeld: 1928
Fermi-Dirac statistics

F. Bloch's theorem: 1928
Bloch electrons

L.D. Landau: 1957
Interacting electrons (Fermi liquid theory)

# Free electron approximation




Neglect periodic potential & scattering (Pauli)

Reasonable for “simple metals” (Alkali Li,Na,K,Cs,Rb)

# Eigenstates & energies

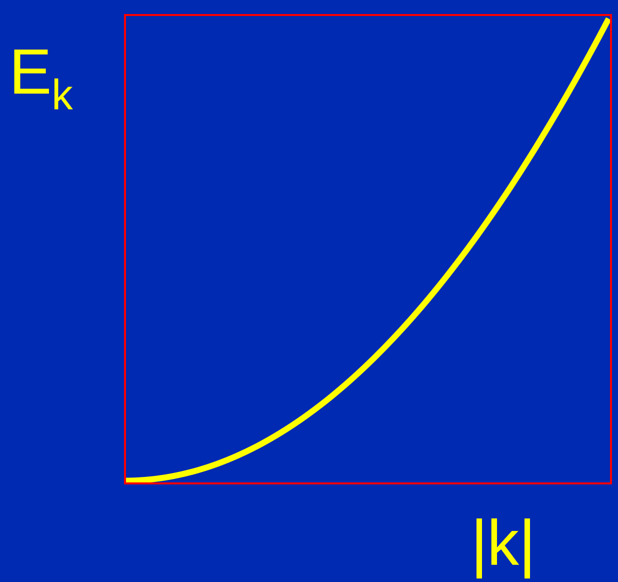
$$\left(\frac{-\hbar^2}{2m}\nabla^2 + \cancel{U}\right)\psi = i\hbar\frac{d\psi}{dt}$$

$$\psi_{\vec{k}}(r,t) = \psi_0 e^{i(\omega t - \vec{k}\cdot\vec{r})}$$

$$E_{\vec{k}} = \frac{\hbar^2}{2m}\left|\vec{k}\right|^2$$

$$\vec{k} = 2\pi(n_x/L_x, n_y/L_y, n_z/L_z)$$

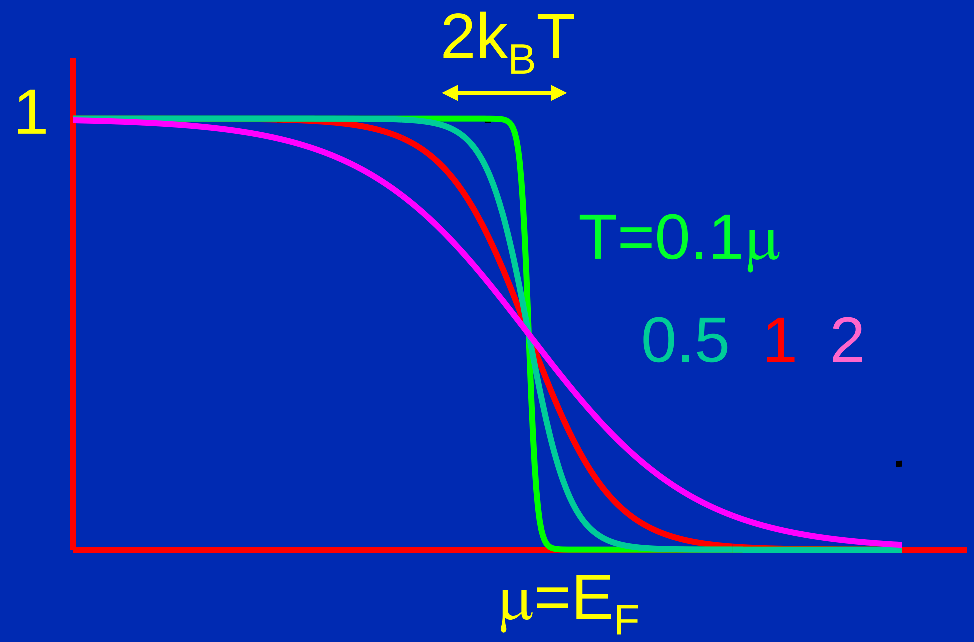
$$D_d(k) \propto L^d \cdot k^{d-1}$$



# Statistics & DOS

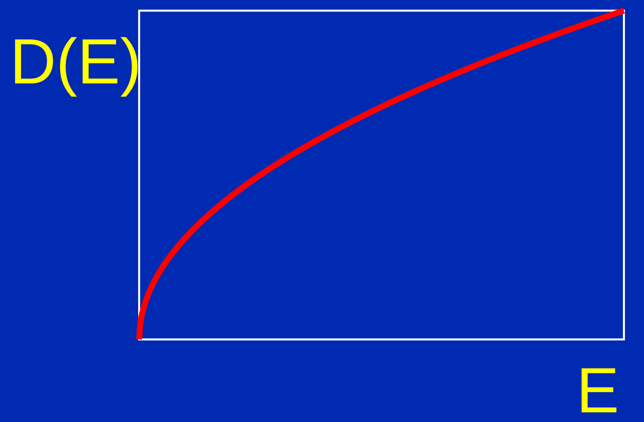
Fermi-Dirac statistics:

$$f_{FD}(E) = \frac{1}{e^{\frac{E-\mu}{kT}} + 1}$$

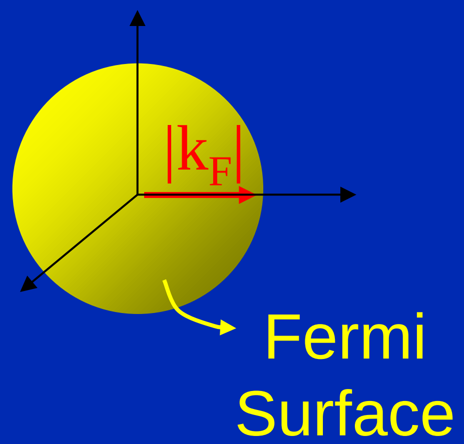


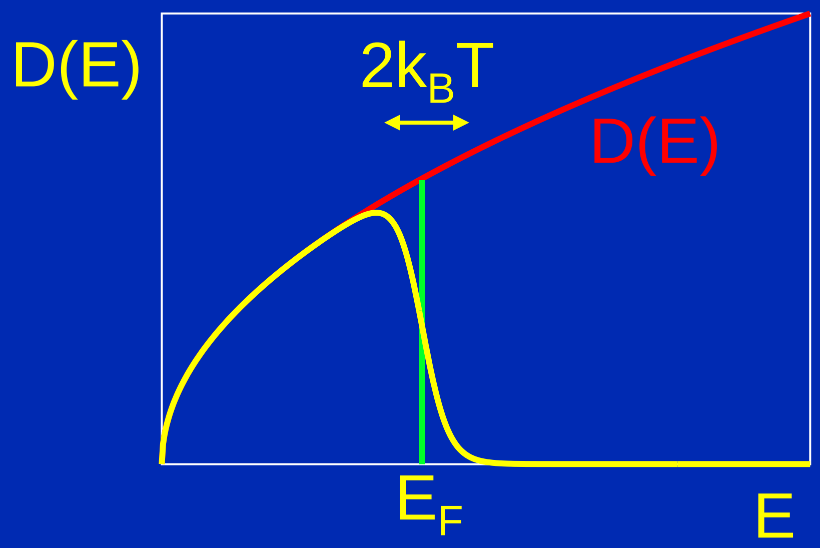

Density of states:

$$D(E) = 2 \cdot D(k) \cdot \frac{dk}{dE} = \frac{Vm}{\pi^2 \hbar^3}\sqrt{2mE}$$



# Occupation of states



# Free electron gas parameters

$$N = \int_0^\infty D(E) \cdot f_{FD}(E)\, dE$$

$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$$

$$E_F = \frac{\hbar^2 k_F{}^2}{2m}$$

$$k_F = \left(3\pi^2 n\right)^{1/3}$$

$$E_F = \frac{1}{2} m v_F{}^2$$

$$v_F = \frac{\hbar k_F}{m}$$

$$T_F = E_F / k_B$$

$$T_F = \frac{\hbar^2}{2mk_B}\left(3\pi^2 n\right)^{2/3}$$

$$D(E_F) = \frac{Vm}{\pi^2 \hbar^3}\sqrt{2mE_F}$$

$$D(E_F) = \frac{V}{\pi\hbar^2} \cdot \left(\frac{3}{\pi} n\right)^{1/3} m$$

# Sodium

Na $1s^2 2s^2 2p^6 3s$ = [Ne]3s

$r_{ion}$ = 0.98 Å $d_{nn}$ = 1.83 Å

$$n = 2.65 \cdot 10^{22}\ \mathrm{cm}^{-3}$$

$$E_F = 3.23\ \mathrm{eV}$$

$$k_F = 2.65 \cdot 10^{22}\ \mathrm{cm}^{-1}$$

$$v_F = 1.07 \cdot 10^{8}\ \mathrm{cm/s}$$ → Non relativistic

$$T_F = 3.75 \cdot 10^{4}\ \mathrm{K}$$ → Degenerate quantum gas
Chemical potential
hardly depends on T

# So far

- Free electrons, i.e. no periodic potential
- Independent electrons, i.e. no $e^-$ - $e^-$ interactions
- Relaxation time approximation ( scattering time $\tau$ )
- Classical statistics (Drude)
- Fermions (Sommerfeld)

→
- Density of states (1D, 2D, 3D)
- Fermi energy ($E_F$, $k_F$ $v_F$, $T_F$)

# t.b.d.

- Sommerfeld (=Drude+quantum)
  - Compressibility
  - Heat capacity
  - Conductivity, Hall effect, 1D conduction
  - Thermal conductivity, Wiedemann-Franz law
- FAILURES of the free electron models
- Including the periodic potential the $e^-$ live in.

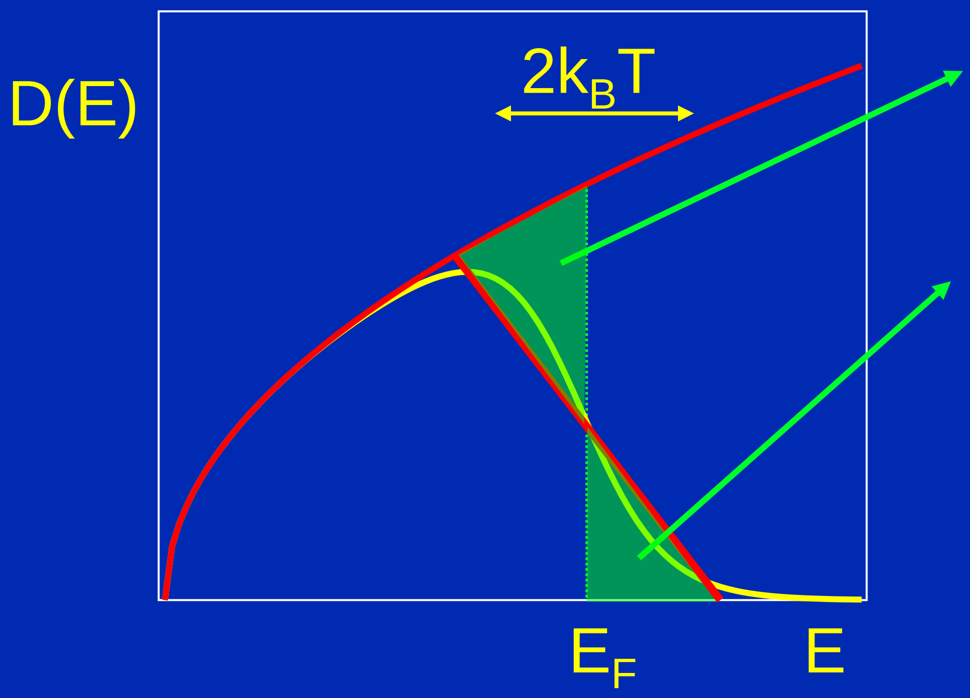
# Compressibility

$$B = 1/K = -V\, dP/dV = V\, d^2U/dV^2$$

| | $B_{f.e.}$ (Gpa) | $B_{obs}$ (Gpa) |
|---|---|---|
| Li | 24 | 12 |
| Na | 9 | 6.5 |
| K | 3 | 3 |
| Rb | 2 | 2 |
| Cs | 1.5 | 1.5 |
| Cu | 64 | 134 |
| Ag | 35 | 100 |
| Al | 228 | 76 |

# Heat capacity: Quick&Dirty

$$C_{el} = \frac{dU_{el}}{dT} = \frac{d}{dT}\int_0^{\infty} E \cdot D(E) \cdot f_{FD}(E,T)\, dE$$




$$-\frac{1}{2}k_B T \times \frac{1}{2} D(E_F) \cdot (E_F - \frac{k_B T}{2})$$

$$+\frac{1}{2}k_B T \times \frac{1}{2} D(E_F) \cdot (E_F + \frac{k_B T}{2})$$

$$\Delta U = \frac{k_B^{\,2} T^2}{2} D(E_F)$$

$$C_{el} = k_B^{\,2} D(E_F) \cdot T$$

$$\Rightarrow \mathbf{C_{el} \propto T}$$

# Heat capacity

Electronic contribution

$$C_{el} = \frac{1}{3}\pi^2 \cdot D(E_F) \cdot k_B{}^2 T = \frac{1}{2}\pi^2 N k_B \frac{T}{T_F} << \frac{3}{2} N k_B$$

Electrons + lattice (low T): $C_v = \gamma \cdot T + A \cdot T^3$

$$\gamma = \frac{1}{3}\pi^2 \cdot D(E_F) \cdot k_B{}^2 \propto m$$

$$\frac{\gamma_{exp}}{\gamma_{f.e.}} \neq 1 \quad \Rightarrow \quad m^*_{th} \equiv \frac{\gamma_{exp}}{\gamma_{f.e.}} m_0$$

Periodic potential (band mass)
e-p interaction (polarons)
e-e interaction

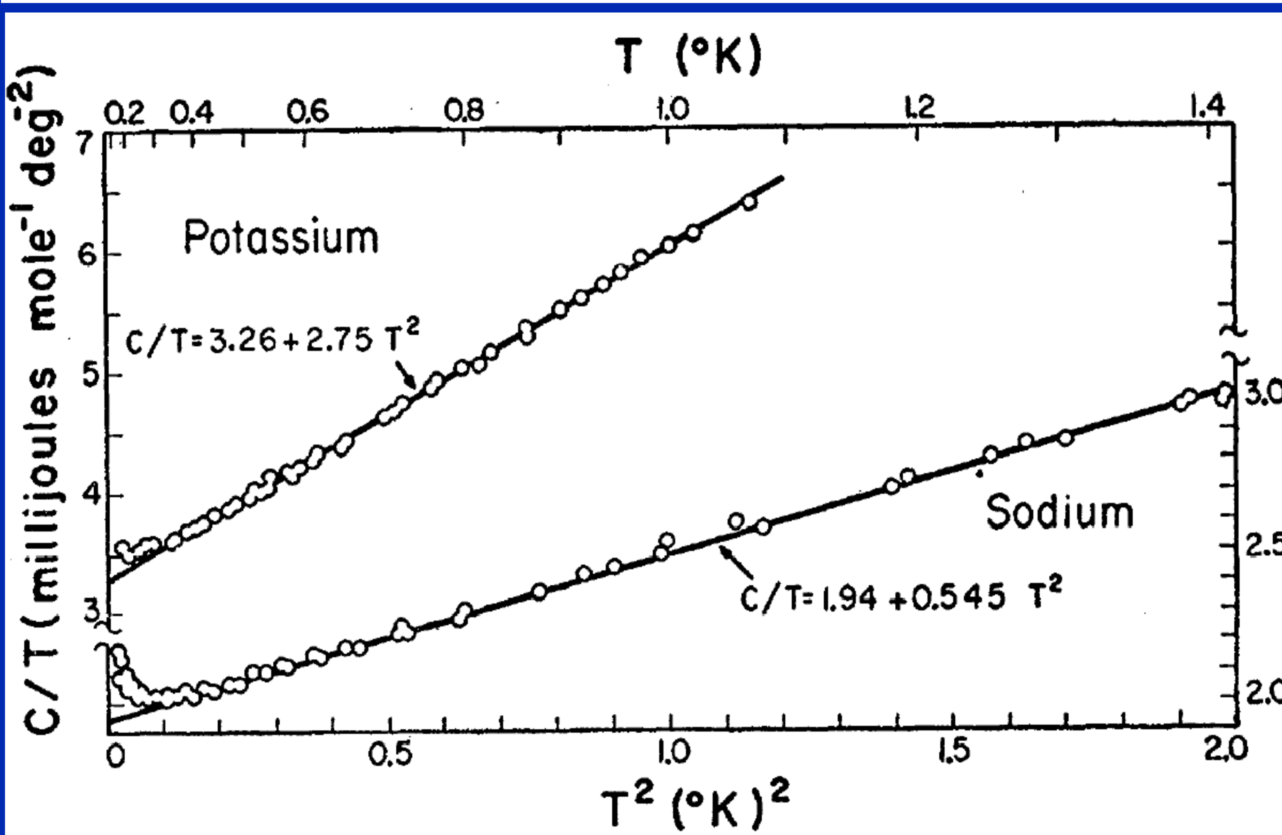
# Heat capacity: Na & K

PHYSICAL REVIEW VOLUME 118, NUMBER 4 MAY 15, 1960

**Heat Capacity of Sodium and Potassium at Temperatures below 1°K**

WILLIAM H. LIEN AND NORMAN E. PHILLIPS
*Department of Chemistry and Lawrence Radiation Laboratory, University of California, Berkeley, California*

(Received December 17, 1959)



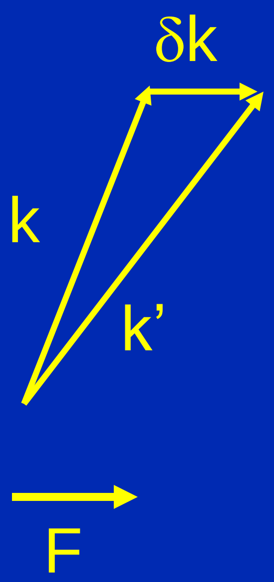

$C = \gamma T + AT^3$

$m_{th} = 1.25\ m_0$

# $\gamma$ & thermal effective mass

| Element | Free $e^-$ $\gamma$ | Expt. $\gamma$ | $m_{th}^*/m_0$ |
|---|---|---|---|
| | $10^{-4}$ cal/mol $K^2$ | | |
| Li | 1.8 | 4.2 | 2.3 |
| Na | 2.6 | 3.5 | 1.3 |
| K | 4.0 | 4.7 | 1.2 |
| Cu | 1.2 | 1.6 | 1.3 |
| Be | 1.2 | 0.5 | 0.42 |
| Fe | 1.5 | 12 | 8 |
| Mn | 1.5 | 40 | 27 |
| Bi | 4.3 | 0.2 | 0.047 |

# Electrical conductivity




2nd Newton: $m\frac{dv}{dt} = F$ or $\delta k = \frac{1}{\hbar}\int F dt$

Relaxation time $\tau$: $\delta k \rightarrow 0$

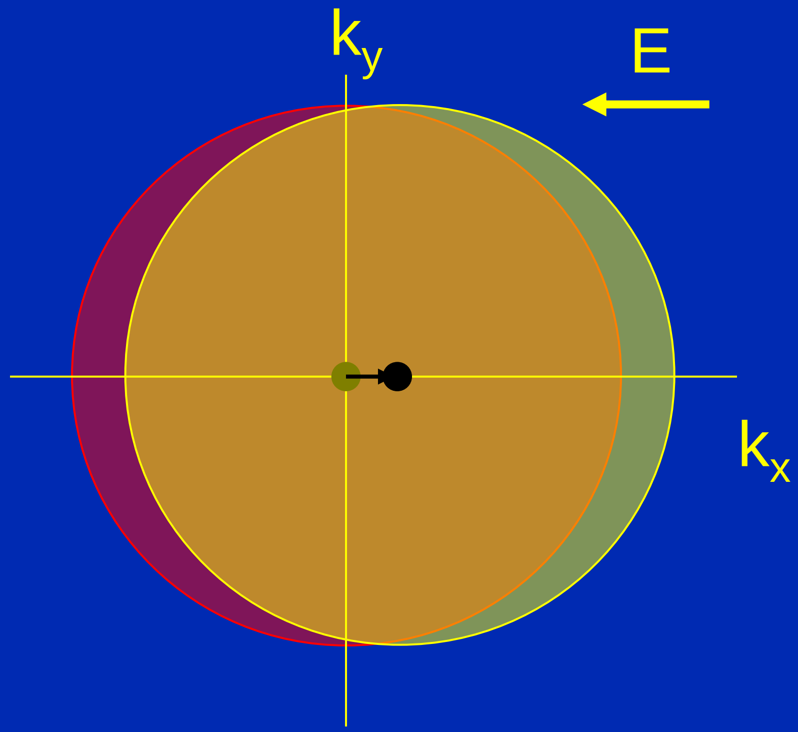
e-p,e-e scattering (appendix J, Ch. 10)
Impurity scattering

$$\delta k = \frac{1}{\hbar} F\tau$$

Equation of motion:

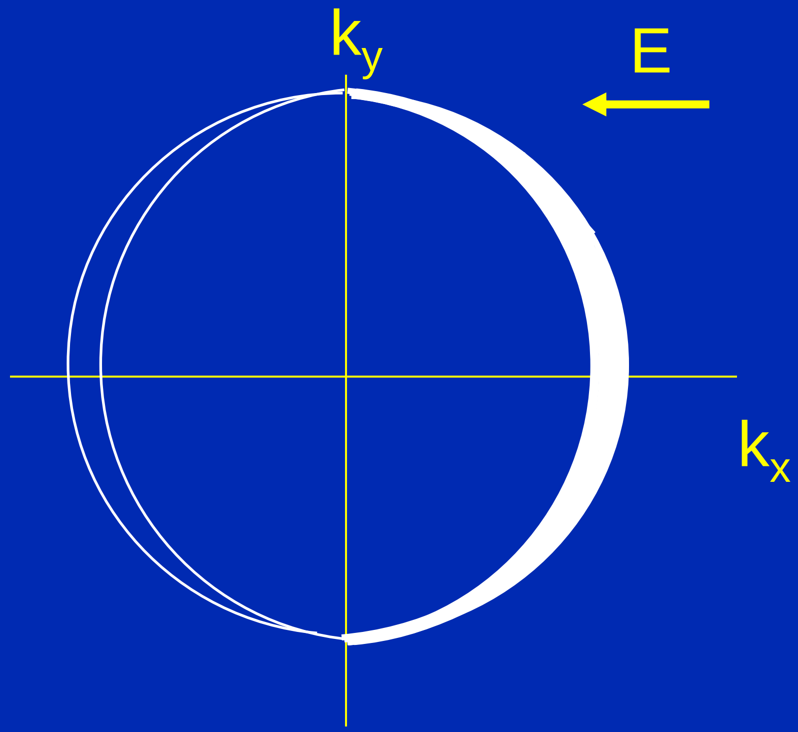
$$\hbar\left(\frac{d}{dt} + \frac{1}{\tau}\right)k = F$$

# Electrical conductivity




$$\delta k = \frac{1}{\hbar} F \tau$$

# Electrical conductivity



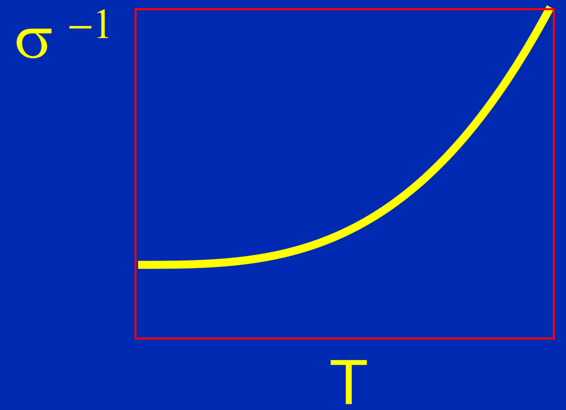

$$\delta k = \frac{1}{\hbar} F\tau$$

# Ohm's law

$$\delta k = \frac{1}{\hbar} F\tau$$

$$F = qE$$

$$\delta k = \frac{qE\tau}{\hbar}; \qquad v_{drift} = \delta v = \frac{qE\tau}{m}$$

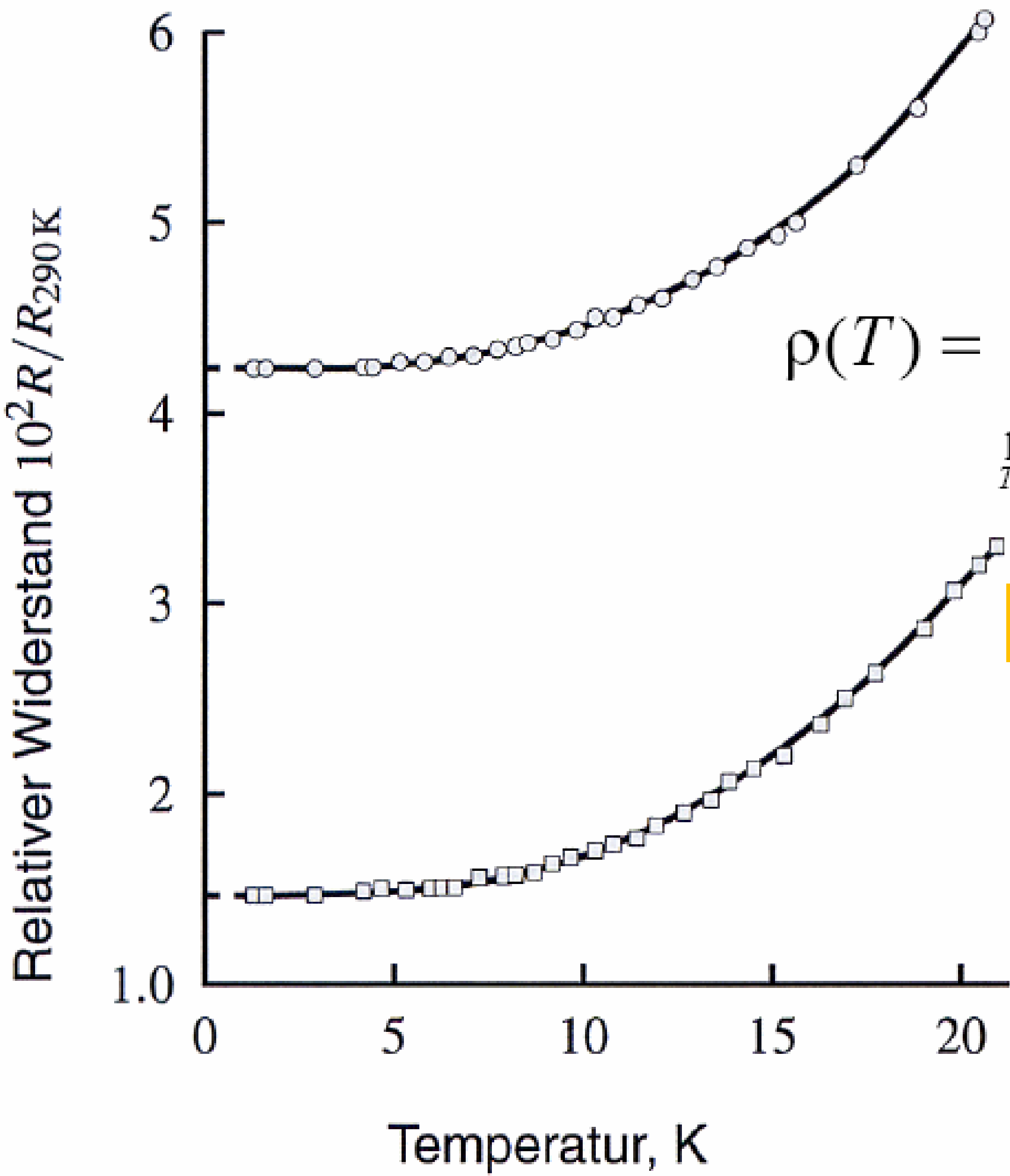



Current density $\quad j = nq\delta v = \dfrac{ne^2\tau E}{m}$

Ohm's law $\quad \sigma = \dfrac{j}{E} = \dfrac{ne^2\tau}{m}$

$$\left.\begin{array}{l} \sigma \sim 10^7\ \Omega^{-1}m^{-1} \\ n \sim 10^{28}\ m^{-3} \end{array}\right\} \quad \tau \sim 10\,fs \quad \Rightarrow \quad \ell = v_F\tau \sim 10\,nm$$

Very pure metals, low T: $\ell > 1\,cm$ !!




Resistance of potassium at 20 K for two different samples.

$$\rho(T) = \underbrace{\rho_{\mathrm{Ph}}(T)}_{\lim_{T\to 0}\rho_{\mathrm{Ph}}(T)=0} + \underbrace{\rho_{\mathrm{i}}(T)}_{\lim_{T\to 0}\rho_{\mathrm{i}}(T)=\rho_{\mathrm{i}}(0)}$$

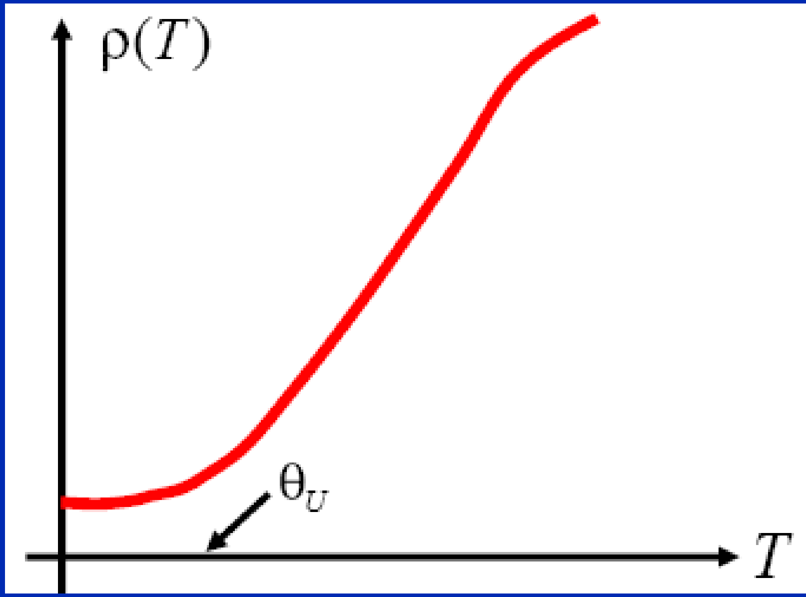
Residual resistance ration RRR

$$\frac{\rho(T = 293\ \mathrm{K})}{\rho(T = 0\ \mathrm{K})} = 1.1\ldots1000$$

An impurity causes a residual resistance of:

$1\cdot 10^{-6}\,\Omega\mathrm{cm}$ pro Atomprozent der Verunreinigung.

## **Temperature dependence of the electric resistance**




(1) Very low temperatures

$\rho_{Phonon} \propto T^5 / \Theta_D^5$

Umklapp-processes are not possible for $T<2K$ in the case of K

(2) Low temperatures

Umklapp-processes become possible

$\langle n \rangle \propto e^{-\Theta_U/T}$ characteristic Umklapp-temperature

Potassium: $\Theta_U = 23K$; $\Theta_D = 91K$

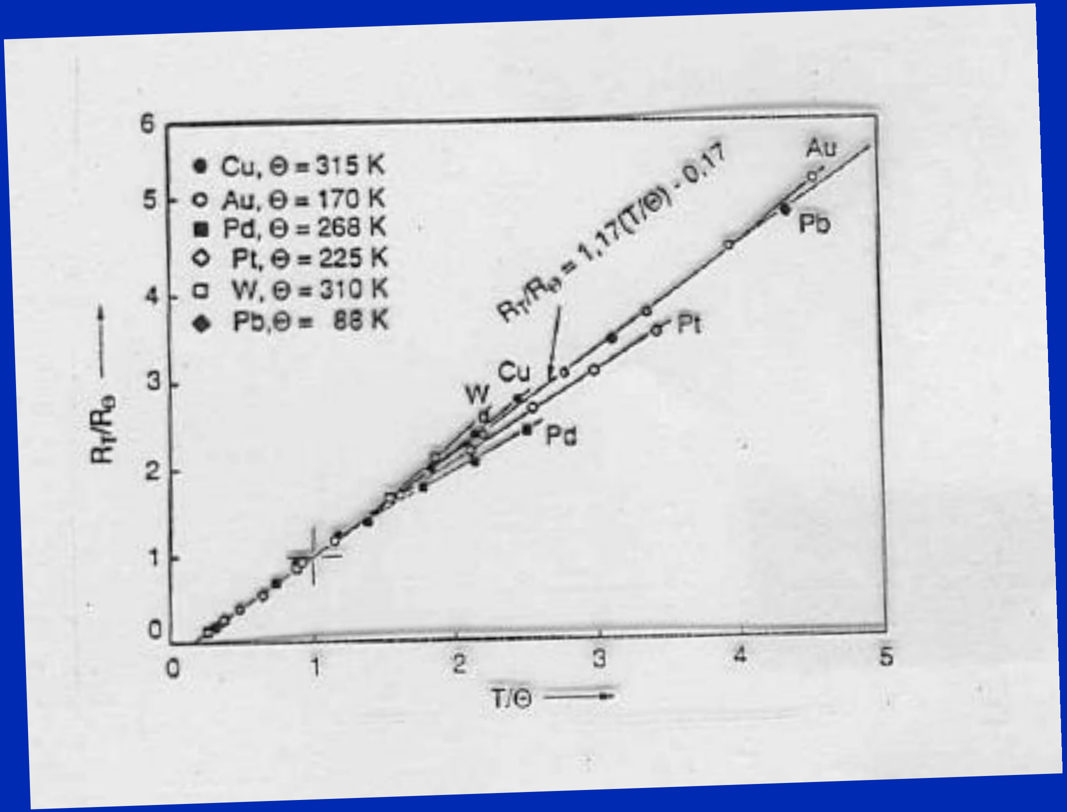
(3) High temperatures

$\langle n \rangle \propto T$ ➔ $\rho_{Phonon} \propto T$

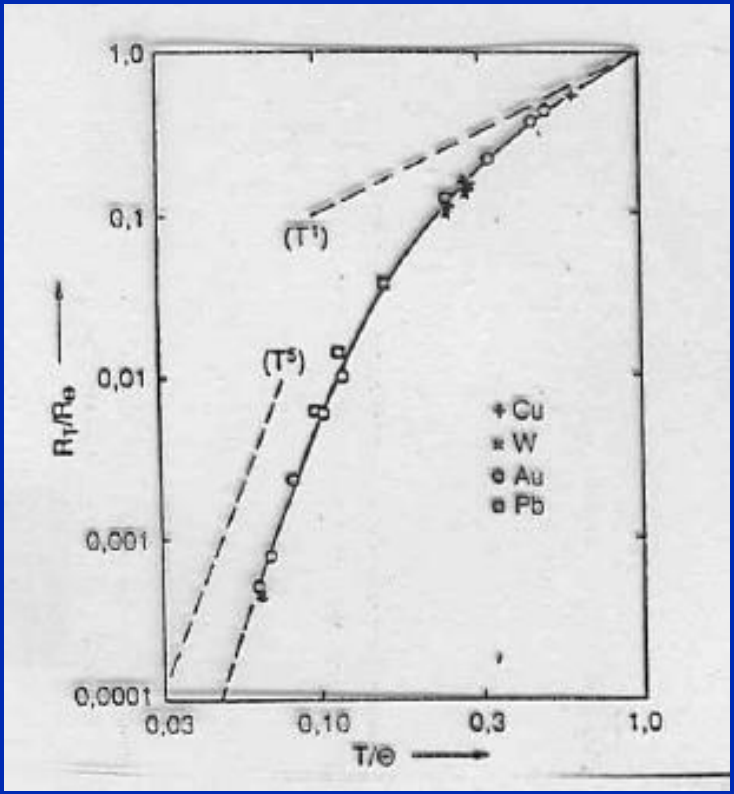
(4) Very high temperatures

saturation as the phonon scattering cannot lead to mean free path-lengths shorter than the lattice constant (Ioffe rule)

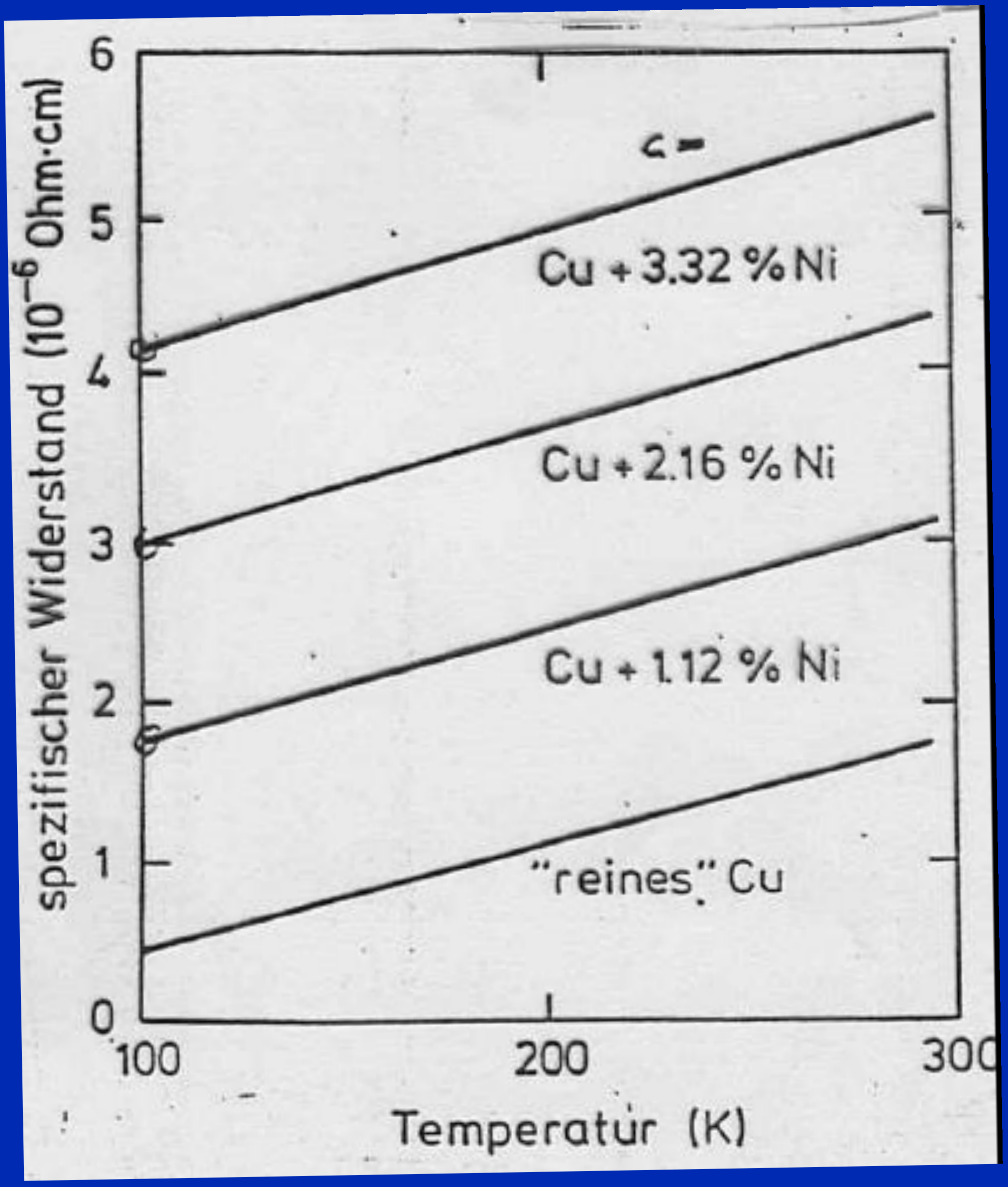
# The resistance roughly scales with the Debye temperature !



# Transition from $T^5$ to T behavior !



# Matthiessen rule: alloys→ vertical displacement of ρ(T) curves



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# Quantized Conductance of Point Contacts in a Two-Dimensional Electron Gas

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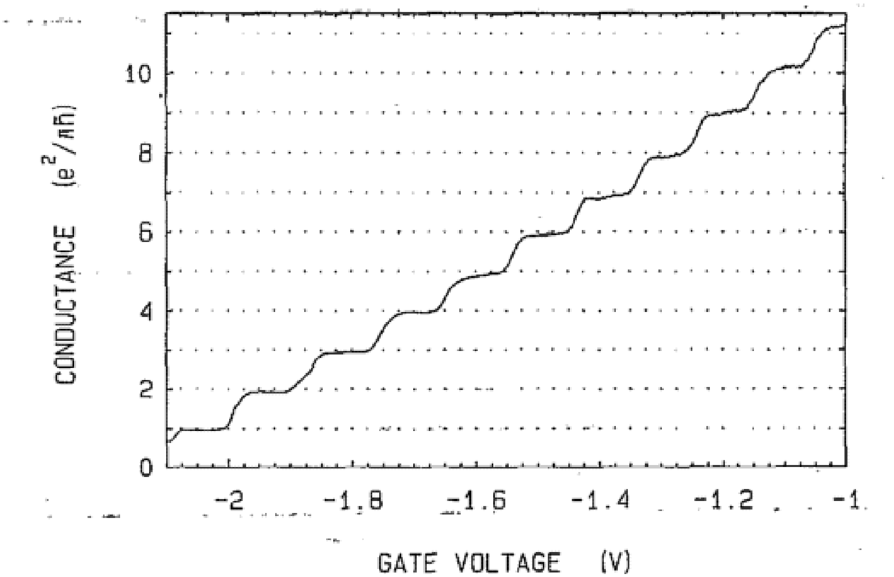


FIG. 2. Point-contact conductance as a function of gate voltage, obtained from the data of Fig. 1 after subtraction of the lead resistance. The conductance shows plateaus at multiples of $e^2/\pi\hbar$.