Condensed Matter Physics I

Prof. Dr. Ir. Paul H.M. van Loosdrecht

II Physikalisches Institut, Room 312

E-mail: pvl@ph2.uni-koeln.de

Website: http:/www.loosdrecht.net/

Previously

Semiconductors:

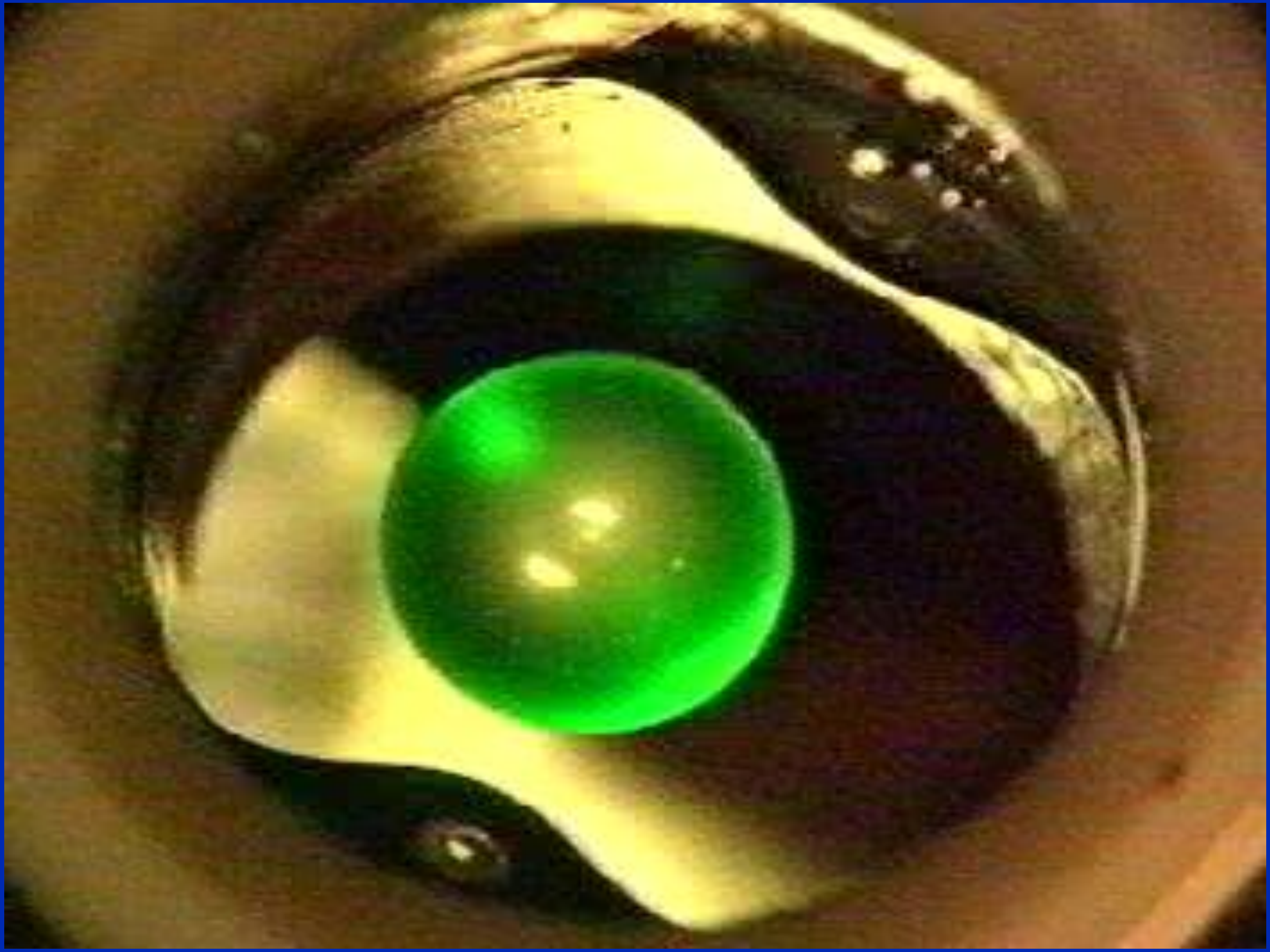
Quantum Hall effect

Today

Introduction magnetism

MAGNETISM

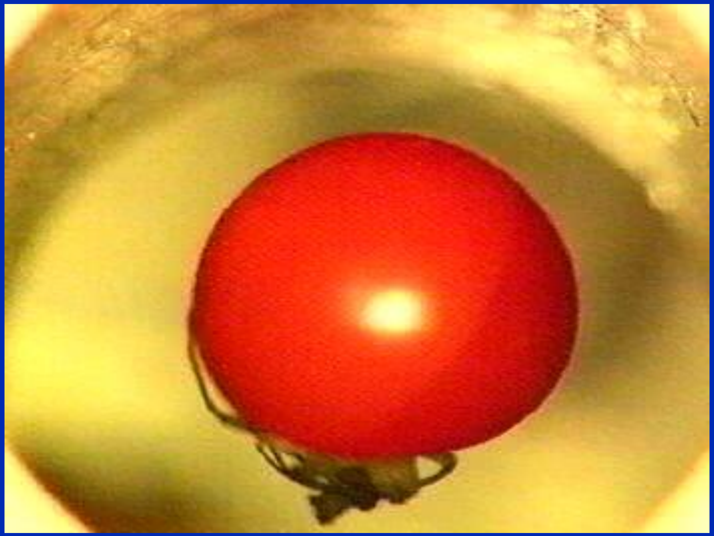
Water shows magnetism



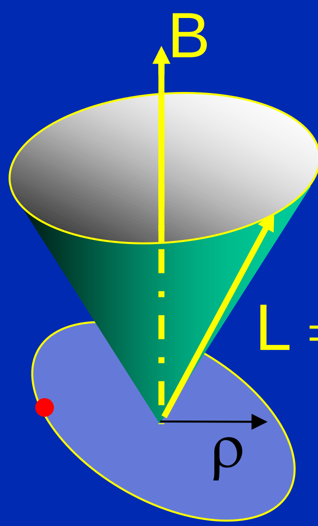
Movies: High magnetic field laboratory Nijmegen

Strawberries do it

Even frogs



‘Classical’ Langevin diamagnetism



e^- in B-field => Larmor precession with $\omega_L = \frac{eB}{2mc}$

Current of Z electrons per atom $I = -Ze \cdot \frac{\omega_L}{2\pi}$

Magnetic moment $\mu = I \cdot A = I \cdot \pi \langle \rho^2 \rangle = -\frac{Ze^2 B}{4mc^2} \cdot \langle \rho^2 \rangle$

Susceptibility of n atoms/volume $\boxed{\chi = \frac{n\mu}{B} = -\frac{nZe^2}{6mc^2} \cdot \langle r^2 \rangle}$

Force: $F = M \frac{\partial H}{\partial x}$

Magnetism

Diamagnetism:

- No magnetic moments
- No magnetic interaction
- Response due to induced currents
- Magnetization <u>opposite</u> to field

- Water
- Ideal gases
- Superconductors

Paramagnetism:

- Magnetic moments (spin, orbit)
- Weak magnetic interactions
- Response due to orientation
- Magnetization in field direction

- Metals
- ‘odd electron’ systems
- O_2, biradicals

Ordered magnetism:

- Magnetic moments
- Strong magnetic interactions
- Response due to polarization
- Ferro-, antiferro-, ferrimagnetic

- Fe, Ni, Co
- Cr, high-T_c (CuO systems)

Magnetization and susceptibility

Magnetization at T=0: $M(H) = -\frac{\partial E_o}{\partial H}$

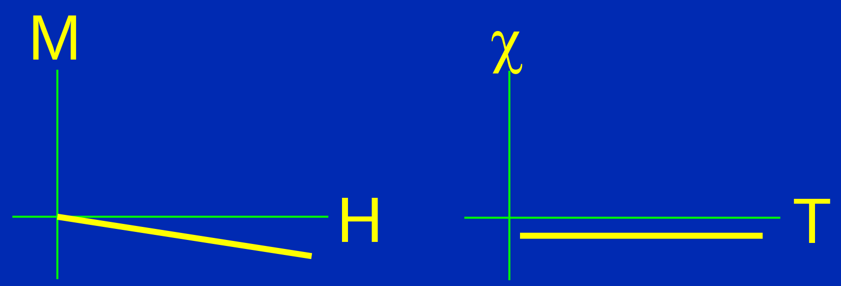
at finite T: $M(H) = \frac{\sum_n M_n(H) e^{-E_n/kT}}{\sum_n e^{-E_n/kT}}$

Magnetic susceptibility: $\chi = \frac{\partial M}{\partial H}$

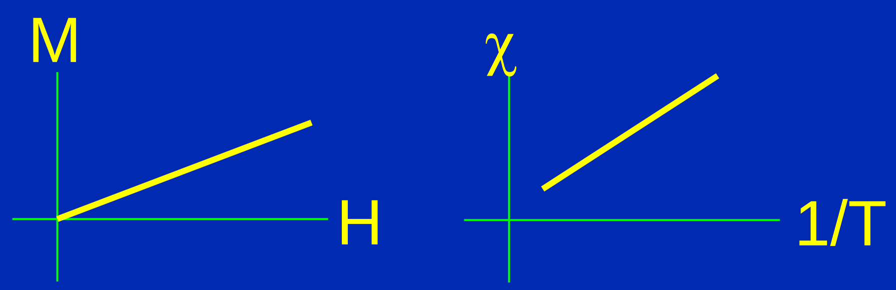
Only ground state (low T): $\chi = -\frac{\partial^2 E_o}{\partial H^2}$

Dia- & paramagnetism

$M = \chi H$



Diamagnetism
Temperature independent



Paramagnetism
1/T dependence

QM treatment: orbit

Inclusion of the field in the motion: $\vec{p} \rightarrow \vec{p} + \frac{e}{c}\vec{A}$ (app. G)

Uniform H-field: $\vec{A} = -\frac{1}{2}\vec{r} \times \vec{H}$ Gauge: $\vec{\nabla} \cdot \vec{A} = 0$;

$$H = T + V \qquad \hbar\vec{L} = \sum_i \vec{r} \times \vec{p}_i$$

$$T = \frac{1}{2m}\sum_i \left[\vec{p}_i + \frac{e}{c}\vec{A}_i\right]^2 = \frac{1}{2m}\sum_i \left[\vec{p}_i - \frac{e}{2c} r_i \times \vec{H}_i\right]^2$$

$$= T_o + \mu_B \vec{L} \cdot \vec{H} + \frac{e^2}{8mc^2}\sum_i (x_i^2 + y_i^2) H^2$$

QM treatment: spin

Inclusion spin moment: $g_o \mu_B \vec{H} \cdot \vec{S} = g_o \mu_B H S_z$

$$\mathcal{H} = T_o + \mu_B (\vec{L} + g_o \vec{S}) \cdot \vec{H} + \frac{e^2}{8mc^2} \sum_i (x_i^2 + y_i^2)\, H^2 = T_0 + \mathcal{H}_B$$

$$E_n = E_{n,o} + E_B$$

$$E_B = \langle n | H_B | n \rangle + \sum_{n \neq n'} \frac{\langle n | H_B | n' \rangle^2}{E_n - E_{n'}}$$

$$\approx \mu_B \vec{H} \cdot \langle n | \vec{L} + g_o \vec{S} | n \rangle + \sum_{n \neq n'} \frac{\left[\mu_B \vec{H} \cdot \langle n | \vec{L} + g_o \vec{S} | n' \rangle \right]^2}{E_n - E_{n'}} + \frac{e^2}{8mc^2} H^2 \left\langle n \middle| \sum_i \left(x_i^2 + y_i^2 \right) \middle| n \right\rangle$$

↳ Curie ↳ Van vleck ↳ Langevin

- Everything is (dia)magnetic
- Langevin diamagnetism: ‘shielding’ effect (Lenz law)
- Meissner effect in superconductors $\chi = -1$
- QM: inclusion of field

Orbit $\vec{p} \rightarrow \vec{p} + \frac{e}{c}\vec{A}$

Spin $g_o \mu_B \vec{H} \cdot \vec{S} = g_o \mu_B H S_z$

$$H = T_0 + H_B$$

$$H_B = \mu_B(\vec{L} + g_o \vec{S}) \cdot \vec{H} + \frac{e^2}{8mc^2} H^2 \sum_i (x_i^2 + y_i^2)$$

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Para/diamagnetism

$$H = T_0 + H_B$$

$$H_B = \mu_B(\vec{L} + g_o\vec{S})\cdot\vec{H} + \frac{e^2}{8mc^2}H^2\sum_i(x_i^2 + y_i^2)$$

$$E_n = E_{n,o} + E_B$$

$$E_B = \langle n|H_B|n\rangle + \sum_{n\neq n'}\frac{\langle n|H_B|n'\rangle^2}{E_n - E_{n'}}$$

$$\mu_B = \frac{\hbar e}{2mc}$$

$$\approx \mu_B\vec{H}\cdot\langle n|\vec{L} + g_o\vec{S}|n\rangle + \sum_{n\neq n'}\frac{(\mu_B\vec{H}\cdot\langle n|\vec{L} + g_o\vec{S}|n'\rangle)^2}{E_n - E_{n'}} + \frac{e^2}{8mc^2}H^2\left\langle n\left|\sum_i\left(x_i^2 + y_i^2\right)\right|n\right\rangle$$

↳ Curie ↳ van Vleck ↳ Langevin diamagnetism

Langevin diamagnetism

$$E_B \approx \mu_B \vec{H} \cdot \langle n | \vec{L} + g_o \vec{S} | n \rangle + \sum_{n \neq n'} \frac{\left[\mu_B \vec{H} \cdot \langle n | \vec{L} + g_o \vec{S} | n' \rangle \right]^2}{E_n - E_{n'}} + \frac{e^2}{8mc^2} H^2 \left\langle n \left| \sum_i \left(x_i^2 + y_i^2 \right) \right| n \right\rangle$$

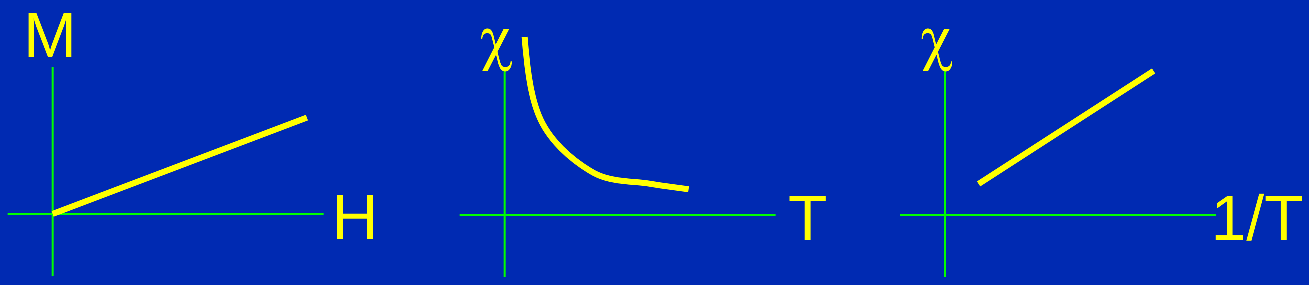
Low temperature, filled shell ions $\left(J|0\rangle = L|0\rangle = S|0\rangle = 0 \right)$

$$E_B \approx \frac{e^2}{12mc^2} H^2 \left\langle 0 \left| \sum_i r_i^2 \right| 0 \right\rangle$$

$$\chi = -n \frac{\partial^2 E_B}{\partial H^2} = -n \frac{e^2}{6mc^2} \left\langle 0 \left| \sum_i r_i^2 \right| 0 \right\rangle = -nZ \frac{e^2}{6mc^2} \langle r^2 \rangle$$

(SI: $\chi = -nZ \frac{\mu_o e^2}{6m} \langle r^2 \rangle$)

Paramagnetism



- alignment of weakly interacting magnetic moments in a magnetic field
- Curie law $\chi = \theta/T$
- Magnetic moments = spin, orbit
- Ground state splitting (Curie)
- Low lying excited states (van Vleck)
- Density of states effects (Pauli magnetism)

First: Coupling between L and S: Russel-Saunders