- response e-gas to inhomogeneous field.

generally one can fourier expand $H(r)$ into cos series
let's take one element of that:

$$\bar{H}(\bar{r}) = \bar{H}_q \cos \bar{q}\cdot\bar{r}$$

that means perturbation $H' = g\mu_B \mu_0 H \cdot S = \pm \frac{g\mu_B \mu_0 |\bar{H}_q|}{2} \cos \bar{q}\cdot\bar{r}$

$\uparrow$ spin up/down.

free electron gas: wave functions

$$\psi_{\bar{k},\sigma} = \frac{1}{\sqrt{V}} e^{i\bar{k}\cdot\bar{r}} \cdot |\sigma\rangle \qquad E_k = \frac{\hbar^2 k^2}{2m}.$$

pert. theory: $$\Psi_{k,\sigma} = \psi_{k,\sigma} + \sum_{\substack{k' \neq k \\ \sigma' \neq \sigma}} \frac{\langle \psi_{k'\sigma'} | H' | \psi_{k,\sigma} \rangle}{E^0_{k\sigma} - E^0_{k'\sigma'}} |\psi_{k'\sigma'}\rangle$$

$$= \frac{1}{\sqrt{V}} \left(e^{ik\bar{r}} \pm \frac{g\mu_0\mu_B H_q}{4} \left[\frac{e^{i(\bar{k}+\bar{q})\bar{r}}}{E_{k+q} - E_k} + \frac{e^{i(\bar{k}-\bar{q})\bar{r}}}{E_{k-q} - E_k} \right] \right) |\sigma\rangle$$

(matrix element only nonzero if $\sigma = \sigma'$ ($\perp$ argument)
and $\int e^{-ik'r} \frac{e^{iqr} + e^{-iqr}}{2} e^{ikr} dr \neq 0$.
i.e. $k' = k+q$ for first e^{iqr}.
$k' = k-q$ for second e^{-iqr}.
so $k' = k \pm q$ only terms in sum).

$|\Psi_k|^2$?
~~taylor expansion of wavefunction~~, keep only terms leading order in H_q.

$$|\Psi_{k,\pm}(r)|^2 = \frac{1}{V} \left(1 \pm \frac{g\mu_0\mu_B H_q m}{\hbar^2} \cdot \left\{ \frac{1}{(k+q)^2 - k^2} + \frac{1}{(k-q)^2 - k^2} \right\} \cos \bar{q}\cdot\bar{r} \right)$$

magnetisation: $M(r) = \frac{g\mu_0\mu_B}{2} \left[|\Psi_{k+}|^2 - |\Psi_{k-}|^2 \right]$

$$= \frac{g^2 \mu_0^2 \mu_B^2 m_e H_q}{\hbar^2 V} \cos(qr) \cdot \left[\frac{1}{(k+q)^2 - k^2} + \frac{1}{(k-q)^2 - k^2} \right]$$

This is magn. due to two particular states k, ~~σ~~ $\pm$

for total magn.: need to integrate over states.

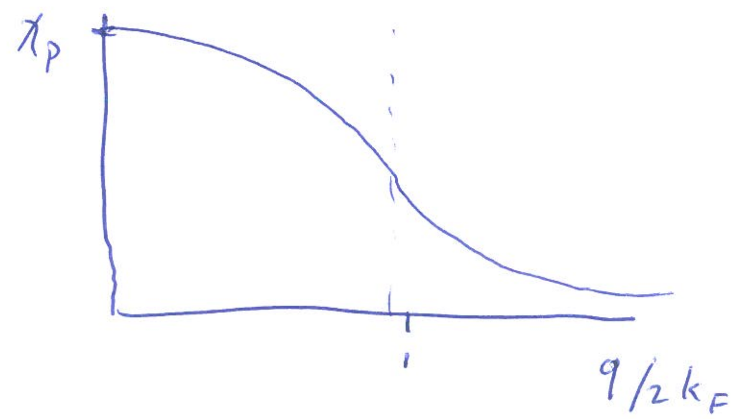
$$M_q = \frac{g^2 \mu_0^2 \mu_B^2 m_e H_q}{\hbar^2 V} \int g(k) \cdot d^3k \cdot \left[\frac{1}{(k+q)^2 - k^2} + \frac{1}{(k-q)^2 + k^2} \right]$$

$$\overset{T=0}{=} \frac{k_F m_e g^2 \mu_0^2 \mu_B^2 m_e H_q}{\pi^2 \hbar^2} \left[1 + \frac{4k_F^2 - q^2}{4k_F q} \cdot \ln \left| \frac{q+2k_F}{q-2k_F} \right| \right]$$

$$\left(\text{using} \int_{k<k_F} \frac{d^3k}{(k+q)^2 - k^2} = \cancel{\int_0^{k_F} 2\pi k^2 dk \int_0^{\pi} \frac{\sin\theta}{q(q-2k\cos\theta)}} \right.$$
$$\left. \frac{\pi k_F}{2} \left[1 + \frac{4k_F^2 - q^2}{4 k_F q} \ln \left| \frac{q+2k_F}{q-2k_F} \right| \right] \right).$$

$$\Rightarrow \chi_q = \frac{M_q}{H_q} = \chi_p \cdot f\left(\frac{q}{2k_F}\right).$$

$$f(x) = \frac{1}{2} \left(1 + \frac{1-x^2}{2x} \ln \left| \frac{x+1}{x-1} \right| \right).$$



$$\left(\chi_p = \frac{3}{2} \frac{n \mu_0 \mu_B^2}{kT_F} \right).$$

now assume.

$$H(r) = \delta(r)\, H = \left(\frac{1}{2\pi}\right)^3 \cdot \underbrace{\int H_q\, e^{iqr}\, d^3q.}_{\text{all freq. components there! \& eq. strong.}}$$

$$\chi(r) = \frac{1}{(2\pi)^3} \int d^3q\, \chi_q\, e^{iqr} = \frac{1}{(2\pi)^3} \int d^3q\, \chi_P\, f\left(\frac{q}{2k_F}\right) e^{iqr}$$

$$= \frac{1}{(2\pi)^3} \int d^3q\, \frac{\chi_P}{2} \left(1 + \frac{4k_F^2 - q^2}{4k_F q} \ln\left|\frac{q+2k_F}{q-2k_F}\right|\right) e^{iqr}$$

$$= \frac{2k_F^3 \chi_P}{\pi} F(2k_F r)$$

$$F(x) = \frac{-x\cos x + \sin x}{x^4}$$