

# Condensed Matter Physics I

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# Previously

## Thermal expansion

- Lattice anharmonicity

## Heat Capacity

- Dulong-Petit law
- Density of states
- Debye model ( $T^3$  law)
- Einstein model
- Phase transitions

# Today

- Heat conductivity
- Intro to second quantization
- Metals

# Thermal conductivity



Thermal energy flux:  $J_u = \frac{dw}{dt}$

Thermal gradient:  $\frac{dT}{dx} = \frac{T_r - T_l}{L}$

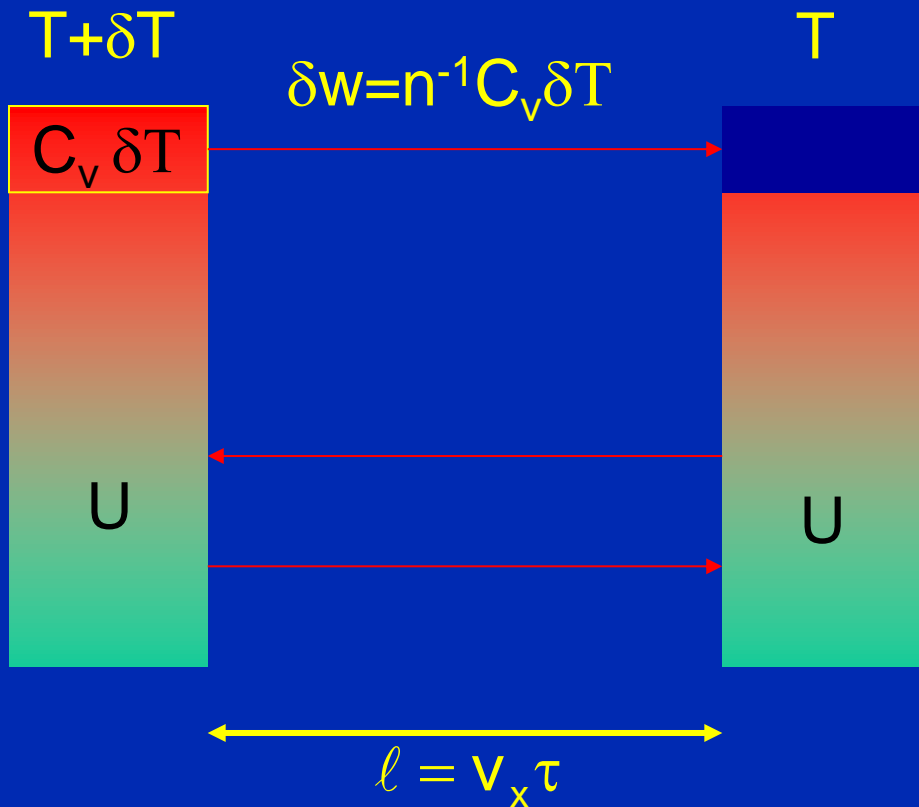
$$J_u = -\kappa \frac{dT}{dx}$$

$\kappa$ : Thermal conductivity

Is due to diffusion of particles (here phonons)

# Thermal conductivity

$$J_u = -\kappa \frac{dT}{dx}$$



$$J_u = \frac{dw}{dt} = n \langle v_x \cdot \delta w \rangle = C_v \langle v_x \delta T \rangle$$

$$\delta T = -v_x \tau \cdot \frac{dT}{dx}$$

$$J_u = -C_v \tau \frac{dT}{dx} \langle v_x^2 \rangle$$

$$\left. \begin{aligned} \langle v_x^2 \rangle &= \frac{1}{3} v^2 \\ \tau &= l / v \end{aligned} \right\} \Rightarrow J_u = -\frac{1}{3} C_v v l \frac{dT}{dx}$$

$$\Rightarrow \kappa = \frac{1}{3} C_v v l$$

# Scattering of phonons

Phonon mean free path  $\ell$  limited by

- Imperfections, crystal boundaries, isotopes
- Phonon-phonon scattering (normal processes)
- Phonon-phonon scattering (Umklapp processes)

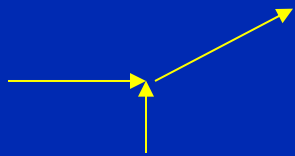
Harmonic approximation in perfect lattice:

Phonons are eigen-states of Hamiltonian  
NO phonon-phonon scattering

# Scattering of phonons

Imperfections etc.:  $l = T$  independent:  $\kappa \propto v l C_V(T) \propto T^3$

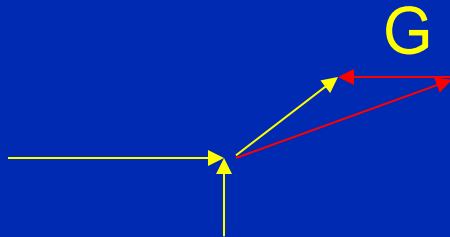
Normal processes:  $\hbar\omega_{k_1} + \hbar\omega_{k_2} = \hbar\omega_{k_3}$



$$\vec{k}_1 + \vec{k}_2 = \vec{k}_3$$

No net change of phonon momentum

Umklapp processes:  $\hbar\omega_{k_1} + \hbar\omega_{k_2} = \hbar\omega_{k_3}$



$$\vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{G}$$

Momentum transferred to crystal:  $\hbar\vec{G}$

# Thermal conductivity

$$\kappa = \frac{1}{3} C_v v \ell$$

High T:  $C_v$  constant, scattering  $\propto n(\omega) \propto T \longrightarrow \kappa \propto T^{-1}$

Low T: Umklapp needs phonon at BZB ( $\sim$ Debye energy)

Very low T: no phonons for U-process:

$\ell$  constant (imperfections)  $\longrightarrow \kappa \propto T^3$

Near  $T \sim T_D$  U-processes become important

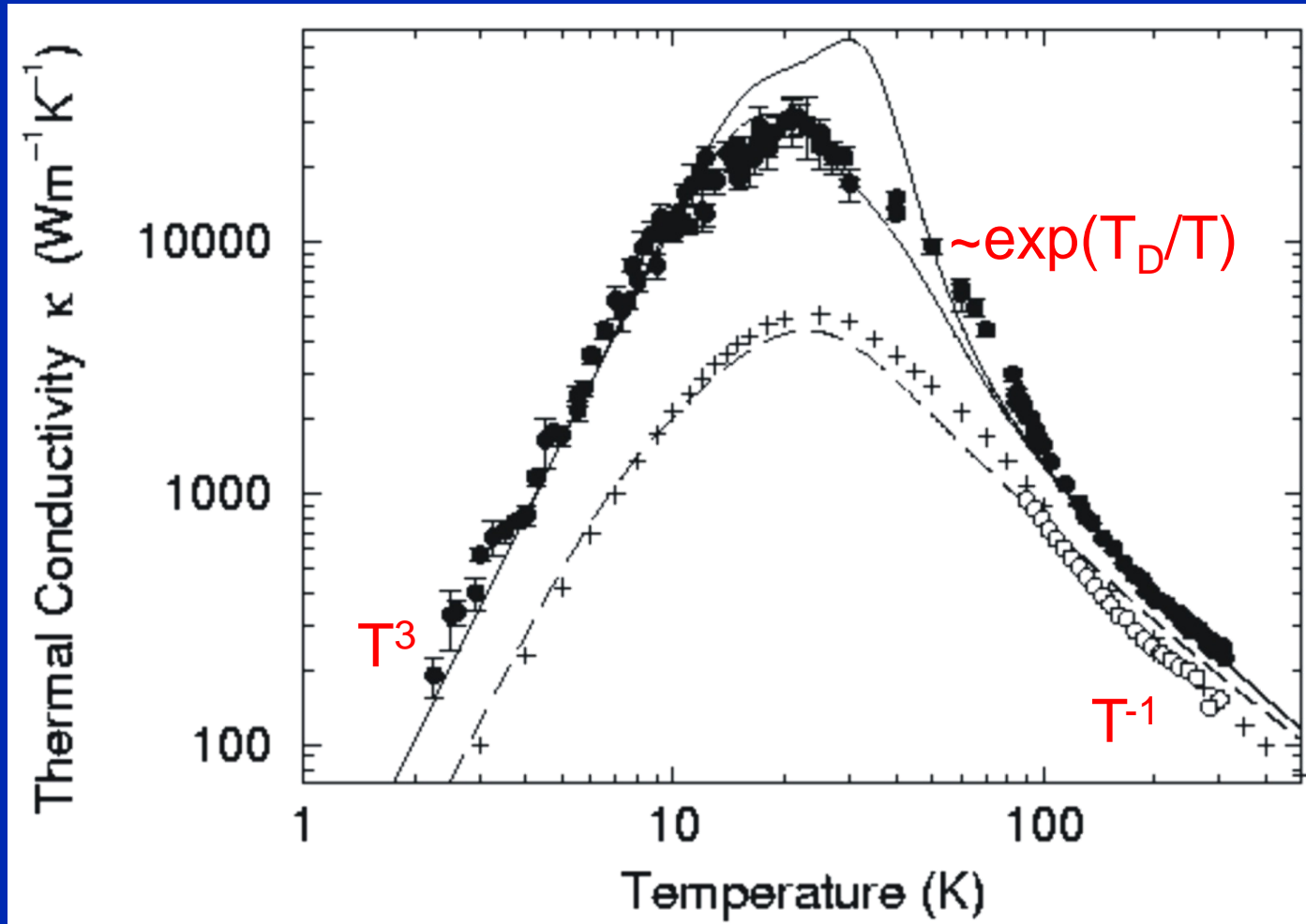
Scattering  $\sim$  # phonons with  $\sim$ Debye energy

$$\ell^{-1} \propto n(\hbar\omega_D/2) = \frac{1}{e^{\theta_D/2T} - 1} \approx e^{-\theta_D/2T}$$

$$\kappa \propto e^{\theta_D/2T}$$



# Thermal conductivity



Isotope pure Silicon (T. Ruf et al. Sol. St. Comm. 2000)