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WS14/15 Condensed Matter Physics I
Exercise 7. Magnetism - I.

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Notice: *In solving the proposed exercises clearly motivate the passages to reach the result. The use of clear and compact notation is greatly encouraged, as well as the systematic use of dimensional checks of the expressions and results. When you are asked to "evaluate" something this means to provide a numerical evaluation of the expression. In this case, at times, it might be necessary to indicate a parameter whose explicit numerical value is not provided, i.e.* $\omega_c = 1.76\,H$ *(Gauss) Hertz. Otherwise specified, all the evaluations are to be given with 3 significant figures.*

7.1 Diamagnetism of harmonic oscillators… (10 pts)

Let us consider ensemble of N one-dimensional spinless particles of mass m and charge q in a volume V. The particles are in a harmonic potential. Let us disregard mutual interactions between particles.

1. Write the total energy of the system in the ground state (i.e. at T=0).
2. Now evaluate the energy at a finite temperature by applying the appropriate formula $E(T) = \sum_n E_n e^{-\beta E_n} / \sum_n e^{-\beta E_n}$.
3. Consider now a homogeneous magnetic field H applied to the system. Evaluate the energy of the system as a function of the temperature. Which field-dependent term in the field-dependent Hamiltonian is non-vanishing in this case? And in three dimensions? Motivate your answers.
4. Evaluate the total magnetization and susceptibility as a function of temperature.

[Hint: Remember the virial theorem for the harmonic oscillator. In evaluating the thermal average It may be useful to use the harmonic series: if $|a| < 1$ $\sum_n a^n = \frac{1}{1-a}$*. Moreover, in this particular case the numerator of the thermal average in point 2 reduces to a particular formal derivative of the denominator. Try to derive the denominator with respect to the inverted temperature.]*

7.2 …and of Hydrogen (10 pts)

Consider a gas of Hydrogen atoms of density n=10^{14} cm^{-3}.

1. Show that in the ground state $\langle r^2 \rangle$ =$3a_0^2$. Remember that the 1s wavefunction is
$$\Psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$
2. Calculate the diamagnetic susceptibility in the ground state.
3. Calculate the paramagnetic spin susceptibility at T=100 K.
4. Does it make sense to compare the 0 K diamagnetic susceptibility with the T=100 K paramagnetic susceptibility? Motivate your answer.

[Hint: use atomics units. Which are the atomic units of the susceptibility?]

7.3 Quantum Paramagnetism (10 pts)

Consider an atom with nonzero magnetic moment in a homogeneous magnetic field along the z direction, described by the Hamiltonian

$$\boldsymbol{H} = -\mu_B g \boldsymbol{J_z} H.$$

Here g is the Lande factor and $\boldsymbol{J_z}$ Is the projection of the total angular momentum of the atom along the field direction. Consider the square modulus of the total momentum to be fixed and measured to be in its quantum number J.

1. Find the energy spectrum of the atom.
2. Write down the average magnetic moment of the atom at the temperature T. Present the results in the form $M = \mu B_J(x)$, where x is an appropriate dimensionless parameter. Write down the expression for $B_{1/2}(x)$ and $B_1(x)$.
3. Now consider J integer and sum the geometric series to obtain an explicit form for B. Compare the sum for J=1/2 with the result obtained in the preceding point. Are they the same? Motivate your answer.
4. Find the magnetic susceptibility per atom.

[Hint: $\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{1-a}$.]