Universität zu Köln
II. Physikalishes Institut
dr. Matteo Montagnese, prof. dr. Paul H.M. van Loosdrecht

WS14/15 Condensed Matter Physics I
Exercise 8. Magnetism -II.

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Notice: *In solving the proposed exercises clearly motivate the passages to reach the result. The use of clear and compact notation is greatly encouraged, as well as the systematic use of dimensional checks of the expressions and results. When you are asked to "evaluate" something this means to provide a numerical evaluation of the expression. In this case, at times, it might be necessary to indicate a parameter whose explicit numerical value is not provided, i.e.* $\omega_c = 1.76\,H$ *(Gauss) Hertz. Otherwise specified, all the evaluations are to be given with 3 significant figures.*

8.1 Small Heisenberg Model. (10 pts)

Consider a Heisenberg model contain a chain of only two spins, so that

$$\mathcal{H} = -J\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

1. Supposing these spins have $S = 1/2$, calculate the energy spectrum of the system. *[Hint: write the Hamiltonian as a function of the total spin $\boldsymbol{S}_1 + \boldsymbol{S}_2$ of the system.]*
2. Consider now three spin forming a triangle. Write down the Hamiltonian and find the spectrum of the system.
3. Do the same now four spins forming a square.

For all the three cases Discuss the number of states, the degeneracy of the states and what happens changing the sign of J.

8.2 Free electron ferromagnetic orderding. (10 pts)

Consider two electrons in a box of volume V. Neglecting coulomb interaction the Hamiltonian is (in atomic units)

$$\mathcal{H} = \frac{1}{2}\nabla_1^2 + \frac{1}{2}\nabla_2^2.$$

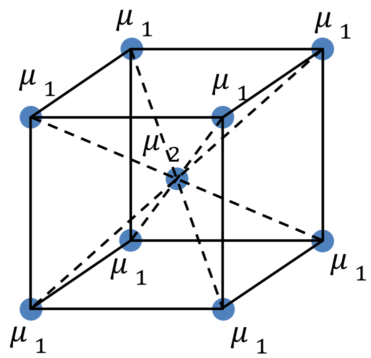
As in the case of the hydrogen molecule, we can take two-electron wave function as a product of the single-electron wavefunctions and suitably symmetrize them to take into account Pauli principles. For spin singlet and the spin triplet sates, respectively. We get

$$\Psi(r_1, r_2) = \frac{1}{2\sqrt{V}}\left[e^{ik_1\cdot r_1}e^{ik_2\cdot r_2} \pm e^{ik_1\cdot r_2}e^{ik_2\cdot r_1}\right].$$

1. Show that these wavefunction are indeed eigenstates of the Hamiltonian above. What is the corresponding energy? Discuss the degeneracy with respect to k_1 and k_2.
2. Concentrate on the triplet state and calculate the probability for the electron 1 to be found in the volume element $d\mathbf{r}_1$ and the electron 2 in the volume element $d\mathbf{r}_2$. Does the result depend on the separation between the two volume elements? Comment and interpret the results.

8.3 Dipole interaction in a BCC crystal. (10 pts)

Consider a magnetic moment μ_2 at center of a cube of other magnetic moments μ_2 at the cube corners as shown in the diagram below.



Using the dipole-dipole interaction for each pair of magnetic dipoles

$$E = -\vec{\mu}_2 \cdot \frac{3(\vec{\mu}_1 \cdot \vec{r}) \cdot \vec{r} - r^2\vec{\mu}_1}{r^5},$$

Compare the dipole-dipole energy due to the interaction of the central moment with its 8 nearest neighbors for $\vec{\mu}_2$ along the x axis and for

(a) $\vec{\mu}_1$ along x
(b) $\vec{\mu}_1$ along y
(c) $\vec{\mu}_1$ along z
(d) All $\vec{\mu}_1$ radially inward.

Which of these configurations has the lowest energy?