Last time: BTE

$$\frac{\partial f}{\partial t} + \bar{v}\cdot\bar{\nabla}_r f + \frac{1}{\hbar}\bar{F}\cdot\bar{\nabla}_k f = \left(\frac{\partial f}{\partial t}\right)_{coll.}$$

in relaxation time approx. and steady state ($\frac{\partial f}{\partial t}=0$).

$$\bar{v}\cdot\bar{\nabla}_r f + \frac{1}{\hbar}\bar{F}\cdot\bar{\nabla}_k f = -\frac{\delta f}{\tau_k} \qquad \delta f = f - f_0.$$

we looked at electrical conductivity (Drude $\bar{J} = \underline{\underline{\sigma}}\,\bar{E}$)
and scattering mechanisms (e-e, impurity, e-p)

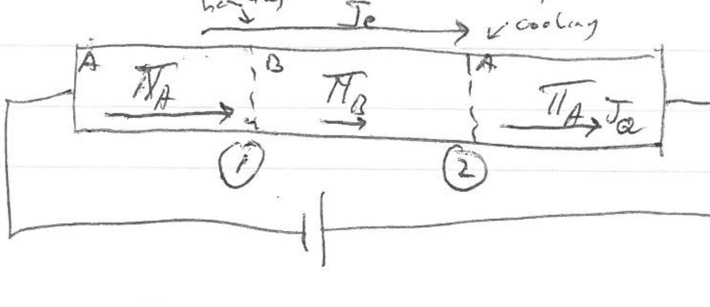
e^- carries - charge
but also - energy (heat, entropy).
~~entropy~~
- spin

through BTE these must be connected!
Electrical transport: $\bar{J}_e = \bar{\bar{\sigma}}\,\bar{E}$
Thermal transport: $\bar{J}_Q = -\bar{\bar{k}}\,\nabla T$

we also know that for instance $\nabla_r T$ (i.e. $T(r)$).
produces electric field. (Thermopower)

$\bar{E} = \bar{\bar{S}}\,\nabla_r T(r)$.
↑ seebeck coefficient

or (2) there is a heat current associated with a thermal current (peltier effect). $J_Q = \Pi J_e$
↑ peltier coeff.



when $\Pi_B < \Pi_A$
@(1) extract heat $(\Pi_A - \Pi_B)J$
@(2) add heat $(\Pi_B - \Pi_A)J$

all these and more are contained in BTE:

~~[illegible]~~ classical particle diffusion.

1. assume $F=0$ (no external fields).
2. assume isotropic, consider x-dir

then BTE in relax. time approx.

$$v_x \frac{\partial f}{\partial x} = -\frac{\delta f}{\tau_s} = -\frac{f-f_0}{\tau_s}$$

$$f = f_0 - \tau_s v_x \frac{\partial f}{\partial x} \simeq f_0 - \tau_s v_x \frac{\partial f_0}{\partial x}.$$

3. For simplicity take classical distribution $f_0 \sim e^{\frac{\mu - \varepsilon}{kT}}$

then $\frac{\partial f_0}{\partial x} = \frac{\partial f_0}{\partial \mu}\frac{\partial \mu}{\partial x} = \frac{f_0}{kT} \cdot \frac{\partial \mu}{\partial x}$.

so that $f \simeq f_0 \left(1 - \tau_s v_x \cdot \frac{1}{kT}\frac{\partial \mu}{\partial x}\right)$

particle flux is then.

$$J_n = \int v_x \cdot D(\epsilon) \cdot f \cdot d\epsilon$$

with $D(\epsilon) = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \cdot \sqrt{E}$
3D D.O.S.

$$= \underbrace{\int v_x D(\epsilon) f_0 \, d\epsilon}_{=0} - \frac{d\mu}{dx}\int v_x^2 \tau_s f_0 \cdot \frac{1}{kT} D(\epsilon) \, d\epsilon.$$

(as many $+v_x$ as $-v_x$).
$\Leftrightarrow$ no net flow in thermal equil.

now assume that $\tau_s \sim \frac{1}{v}$ (i.e. fixed mean free path. $l = v \cdot \tau$).

then.

$$J_n = -\frac{d\mu}{dx} \cdot \frac{l}{kT} \int \frac{v_x^2}{v} \cdot f_0 \, D(\epsilon) \, d\epsilon$$

1 equip. : $v_x^2 = v_y^2 = v_z^2 = \frac{1}{3} v^2$.

2 $\mu = kT \cdot \ln(n) + C$. $\Rightarrow \frac{\partial \mu}{\partial x} = kT \frac{1}{n} \cdot \frac{\partial n}{\partial x}$.

~~using theses:~~

using 1: $\int \frac{v_x^2}{v} f_0 D(\epsilon) d\epsilon = \frac{1}{3} \int v f_0 D(\epsilon) d\epsilon = \frac{1}{3} n \langle v \rangle$

↳ average speed

then using also 2 :

$$J_n = -\frac{1}{3} l \langle v \rangle \cdot \frac{dn}{dx}$$

~~normally~~ Diffussion : $J = -D \cdot \frac{\partial n}{\partial x}$.

$$\Rightarrow D = \frac{1}{3} l \langle v \rangle \quad [m^2/s].$$

More general: we can have
- thermal gradients
- chemical pot. gradients
- applied fields
  ↳ here: static uniform.

def.: electrochemical potential $\eta(r)$.

$$\eta(r) = -e\,\phi(r) + \mu(r).$$

$$\nabla\eta(r) = -e\,\underbrace{\bar{\nabla}_r \phi(r)}_{= -E\text{-field}} + \bar{\nabla}_r \mu(r). = \vec{G}$$

generalized force field

BTE: $$\underset{①}{v_k \bar{\nabla}_r f} + \underset{②}{\frac{1}{\hbar} F \bar{\nabla}_k f} = \left(\frac{\partial f}{\partial t}\right)_{coll.}$$

② : $$\frac{\bar{F}}{\hbar}\frac{\partial f}{\partial k} = \frac{\bar{F}}{\hbar}\frac{\partial f}{\partial \varepsilon}\cdot\frac{\partial \varepsilon}{\partial k} = \bar{F}\,\frac{\partial f}{\partial \varepsilon}\cdot v_k \simeq \bar{v}_k \bar{F}\frac{\partial f_0}{\partial \varepsilon} \quad \left(\frac{\partial \varepsilon}{\partial k} = \hbar v\right)$$

① $$\frac{\partial f}{\partial \bar{r}} = \frac{\partial f}{\partial \mu}\nabla\mu + \frac{\partial f}{\partial T}\nabla T. \quad (\text{both } \mu = \mu(r) \ \& \ T = T(r))$$

ⓐ $$\frac{\partial f}{\partial \mu} \simeq \frac{\partial f_0}{\partial \mu} = \frac{f_0^2}{kT}\, e^{(\varepsilon_k - \mu)/kT}$$

ⓑ $$\frac{\partial f}{\partial T} \simeq \frac{\partial f_0}{\partial T} = \frac{\varepsilon_k - \mu}{(kT)^2}\cdot f_0^2\, k_B\, e^{(\varepsilon - \mu)/kT}$$

ⓒ $$\frac{\partial f}{\partial \varepsilon} \simeq \frac{\partial f_0}{\partial \varepsilon} = -\frac{(f_0)^2}{kT}\, e^{\frac{\varepsilon - \mu}{kT}}$$

using ⓒ : ⓐ and ⓑ become.

$$\frac{\partial f}{\partial \mu} \simeq -\frac{\partial f^0}{\partial \varepsilon}$$

$$\frac{\partial f}{\partial T} \simeq -\frac{\varepsilon - \mu}{T}\cdot\frac{\partial f^0}{\partial \varepsilon}$$

then BTE becomes in RTA:

$$\delta f = -\frac{\partial f_0}{\partial \varepsilon_k} \cdot \tau_k \, \bar{v}_k \cdot \left(\tilde{G} + \frac{\varepsilon_k - \mu}{T} \nabla_r T \right)$$

with $\tilde{G} = \bar{F} - \nabla_r \mu = -e E - \nabla_r \mu$

for homogeneous E-field.

and currents:

$$J_e = -e \int \frac{d^3k}{(2\pi)^3} \, v_k \cdot \delta f$$

$$J_Q = \int \frac{d^3k}{(2\pi)^3} \cdot (\varepsilon_k - \mu) \cdot v_k \cdot \delta f$$

which can be written as

$$J_e = e \hat{K}^{(0)} \cdot G - \frac{e}{T} K^{(1)} (-\nabla_r T)$$

$$J_Q = -K^{(1)} \cdot G + \frac{1}{T} K^{(2)} (-\nabla_r T)$$

$K^{(n)}$: tensor

in isotropic case:

$$K^{(n)} = -\frac{n_0}{m^*} \int d\varepsilon \, \frac{\partial f}{\partial \varepsilon} \tau(\varepsilon) (\varepsilon - \mu)^n$$

for $T \ll T_F$ this becomes

$$K^{(0)}(\varepsilon_F) = \frac{n_0(\varepsilon_F) \, \tau(\varepsilon_F)}{m^*} \qquad K^{(1)}(\varepsilon_F) = \frac{\pi^2}{3} (k_b T)^2 \frac{\partial}{\partial \varepsilon} K^{(0)} \Big|_{\varepsilon_F}$$

and $K^{(2)} = \frac{\pi^2}{3} (k_b T)^2 \cdot K^{(0)}(\varepsilon_F)$.

A) for instance <u>conductivity</u>: $\nabla_r T = 0$.

$$J_e = e k^{(0)} G = \underbrace{e^2 k^{(0)}} E$$

$$\sigma = e^2 k^{(0)} = \frac{n e^2 \tau}{m}.$$

B) thermal cond.: $J_e = 0$. $\Rightarrow$ $\boxed{G = -\frac{k^{(1)}}{T k^{(0)}} \nabla T}$ (1)

$$\Rightarrow J_Q = -k^{(1)} \cdot \left[-\frac{k^{(1)}}{T k^{(0)}} \nabla T \right] + \frac{1}{T} k^{(2)} (-\nabla T)$$

$$= -\frac{1}{T} \Big(k^{(2)} - \underbrace{\frac{k^{(1)\,2}}{k^{(0)}}}_{\text{usually small in metals}} \Big) \nabla T = -\kappa \nabla T$$

$$\Rightarrow \kappa \simeq \frac{1}{T} k^{(2)} = \frac{\pi^2 k_b^2}{3} T k^{(0)} = \frac{\pi^2}{3} \frac{k_b^2}{e^2} T \sigma$$

$\Rightarrow$ Wiedemann-Franz law

$$\frac{\sigma}{\kappa} = \frac{3 e^2}{k_b^2 T \pi^2}$$

C) Thermoelectric effects. $\nabla_r T \Rightarrow E$ see (1)

$$G = S \cdot \nabla_r T = -\frac{k^{(1)}}{T k^{(0)}} \nabla T$$

Seebeck coeff.

$$\Rightarrow S = -\frac{k^{(1)}}{T k^{(0)}}$$

E.g. $qE = S \cdot \nabla_r T$

other TE effect: peltier. ($\nabla T = 0$)

$$J_e = e^2 k^{(0)} E$$

$$J_Q = -k^{(1)} E$$

$$J_q = -\frac{k^{(1)}}{k^{(0)}} J_e = \Pi J_e$$

$$= Q \cdot T \cdot J_e$$

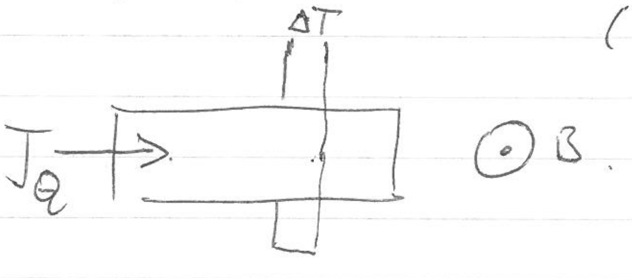
peltier coeff. Π

$$\Pi = Q T$$

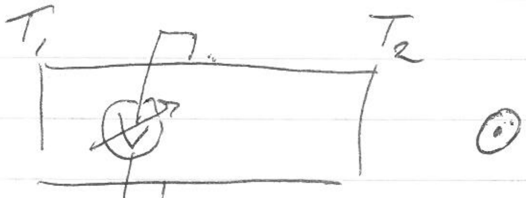
seebeck

In magnetic field: Hall effect. → known.

equivalent ~~pro~~ effect in heat transport.
(Righi-leduc.)



Nernst effects:

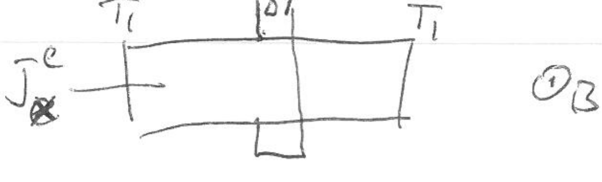


$\partial T/\partial x \neq 0$
$\partial T/\partial y = \partial T/\partial z = 0$
$j_x = j_y = 0$

$$E_y = N \cdot B \cdot \frac{\partial T}{\partial x}$$

~~Righi-Leduc~~

Ettinghausen effect (= nernst-2)



$$\frac{\partial T}{\partial y} = K \cdot \frac{B}{\kappa} j_x$$

heat cond.