16-11-2015 (1)
Supercond. US1516

- Josephson effects.
- squid.

quantum tunneling

1973 Nobel prize 1973.

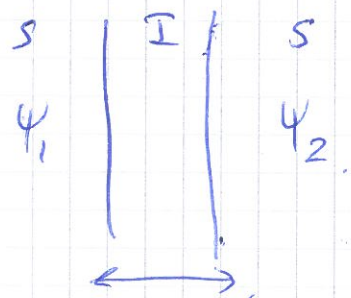
Leo isaki: tunneling in semiconductors
Ivar Giaever: " " superconductors
Brian Josephson: theoretical prediction properties supercurrent through tunnel barriers Josephson effects

Josephson looked at Giaever tunneling for supercond. and noted a dependence on phase of superconducting condensate.

simple treatment: 2 identical superconductors.
macroscopic wfcts.

(1) $\psi_1 = \sqrt{n_1}\, e^{i\theta_1}$

$\psi_2 = \sqrt{n_2}\, e^{i\theta_2}$.



tunneling prob. T $[s^{-1}]$

then change in time of ψ's:

(2) $i\hbar \frac{\partial \psi_1}{\partial t} = \hbar T \psi_2$

$i\hbar \frac{\partial \psi_2}{\partial t} = \hbar T \psi_1$

fill in (1) $\Rightarrow$ $\frac{1}{2}\frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = -iT\sqrt{n_1 n_2}\, e^{i\delta}$

with $\delta = \theta_2 - \theta_1$ $\frac{1}{2}\frac{\partial n_2}{\partial t} + i n_2 \frac{\partial \theta_2}{\partial t} = -iT\sqrt{n_1 n_2}\, e^{-i\delta}$

Scr.

$$i\hbar \frac{\partial}{\partial t} \sqrt{n_1}\, e^{i\theta_1} = \hbar T \sqrt{n_2}\, e^{i\theta_2}$$

$$i\hbar \frac{\partial}{\partial t} \sqrt{n_2}\, e^{i\theta_2} = \hbar T \sqrt{n_1}\, e^{i\theta_1}$$

$$i\hbar\left(e^{i\theta_1}\frac{\partial \sqrt{n_1}}{\partial t} + \sqrt{n_1}\frac{\partial e^{i\theta_1}}{\partial t}\right) =$$

$$i\hbar\left(e^{i\theta_1}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{n_1}}\cdot\frac{\partial n_1}{\partial t} + i\sqrt{n_1}\, e^{i\theta_1}\frac{\partial \theta}{\partial t}\right) = \hbar T \sqrt{n_2}\, e^{i\theta_2}$$

$$\Rightarrow \quad \frac{1}{2}\frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = -iT\sqrt{n_1 n_2}\, e^{i\delta} \qquad \delta = \theta_2 - \theta_1.$$

taking real & imag parts:

$$\frac{\partial n_1}{\partial t} = 2T\sqrt{n_1 n_2}\,\sin\delta \qquad \frac{\partial n_2}{\partial t} = -2T\sqrt{n_1 n_2}\,\sin\delta.$$

$$\frac{\partial \theta_1}{\partial t} = -T\sqrt{\frac{n_2}{n_1}}\cos\delta \qquad \frac{\partial \theta_2}{\partial t} = -T\sqrt{\frac{n_1}{n_2}}\cos\delta$$

identical superconductors $(n_1 = n_2)$.

$$\Rightarrow \quad \frac{\partial \theta_1}{\partial t} = \frac{\partial \theta_2}{\partial t} \quad \Rightarrow \quad \frac{\partial(\theta_2 - \theta_1)}{\partial t} = 0.$$

$$\frac{\partial n_1}{\partial t} = -\frac{\partial n_2}{\partial t}.$$

So if $\delta \neq 0$: current proportional to $\sin\delta$.

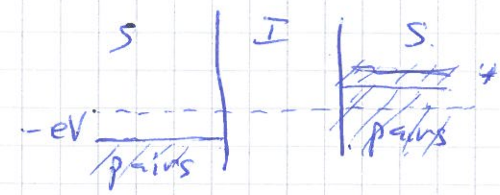
$$\Rightarrow \quad J = J_0 \sin(\theta_2 - \theta_1) \qquad (J_0 \sim T)$$

Note that we did not assume any voltage applied!!

at some point of course $n_1 \neq n_2$ ☺

Now let us apply a voltage.

electron pair ($q=-2e$) ~~charges~~ sees potential qV when crossing barrier, so let's say
pair on left has $-eV$, pair on right $+eV$.



$eV < \Delta$

then ($i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi$)

$$i\hbar \frac{\partial \psi_1}{\partial t} = \hbar T \psi_2 - eV\psi_1$$

$$i\hbar \frac{\partial \psi_2}{\partial t} = \hbar T \psi_2 + eV\psi_2$$

this gives

$$\frac{1}{2}\frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = i\frac{eVn_1}{\hbar} - iT\sqrt{n_1 n_2}\, e^{i\delta}$$

$$\frac{1}{2}\frac{\partial n_2}{\partial t} + i n_2 \frac{\partial \theta_2}{\partial t} = -i\frac{eVn_2}{\hbar} - iT\sqrt{n_1 n_2}\, e^{-i\delta}$$

Imag & real:

$$\frac{\partial n_1}{\partial t} = 2T\sqrt{n_1 n_2}\sin\delta \qquad \frac{\partial n_2}{\partial t} = -2T\sqrt{n_1 n_2}\sin\delta$$

$$\frac{\partial \theta_1}{\partial t} = \frac{eV}{\hbar} - T\sqrt{\frac{n_2}{n_1}}\cos\delta \qquad \frac{\partial \theta_2}{\partial t} = -\frac{eV}{\hbar} - T\sqrt{\frac{n_1}{n_2}}\cos\delta$$

again limit $n_1 = n_2$

$$\frac{\partial \theta_2 - \theta_1}{\partial t} = \frac{\partial \delta}{\partial t} = -\frac{2eV}{\hbar} \quad \text{and} \quad J = J_0 \sin(\delta(t))$$

$$\delta(t) = \delta(0) - \frac{2e}{\hbar}\int V\,dt \quad \text{if } V = \text{constant} = V_0$$

then $\delta(t) = \delta(0) - \frac{2e}{\hbar}V_0 t$

and $J = J_0 \sin\left[\delta(0) - \frac{2e}{\hbar}V_0 t\right]$

Surprising conclusion is that by applying a constant voltage V_0 there is an oscillating current $\left(\omega_0 = \frac{2e}{\hbar} V\right)$

DC voltage of $1\,\mu V \rightarrow 483.6$ MHz

① DC Josephson effect:

DC current flowing in SIS junction without applied voltage.

$$J = J_0 \sin \delta$$

② AC Josephson effect.

AC current flowing in SIS junction when applying constant voltage V_0.

$$J = J_0 \sin(\delta_0 - \omega_0 t).$$

$$\omega_0 = \frac{2e}{\hbar} V_0 . = \frac{2\pi}{\phi_0} V_0 \quad \left(\phi_0 = \frac{h}{2e}\right)$$

$\Rightarrow$ voltage standard: freq. $\frac{2e}{\hbar}$ defines 1V

$\Rightarrow$ tunable microwave "source" $\frac{\omega_0 / 2\pi}{V_0} = \frac{1}{\phi_0} = 484$ GHz/mV

- single junction: very weak power. need many $\Rightarrow$ use high T_c's.

CuO ——— S.C.

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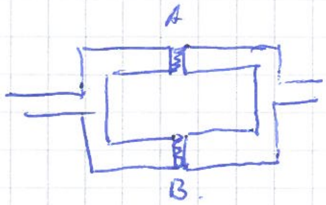
SQUID.



total current : $I_{tot} = I_A + I_B =$

$$I_{max}(\sin\delta_A + \sin\delta_B)$$

$$= 2I_{max}\sin\frac{\delta_A+\delta_B}{2}\cos\frac{\delta_A-\delta_B}{2}$$

maximum if $\delta_A = \delta_B = 0 \underbrace{\pm 2\pi N}$

can we influence this?

yes! flux through ring.

$$\Delta\varphi = \frac{q}{\hbar}\int_{path}\bar{A}\,d\bar{l}$$

phase picked up by charge when moving in vector field.

using stokes theorem

$$\oint \bar{A}\,d\bar{l} = \iint \bar{\nabla}\times\bar{A}\,d\bar{s} = B\cdot\text{Opp.} = \text{flux}.$$

$$(B = \nabla\times A)$$

two cooper pairs arriving at exit through diff. paths:

$$\Delta\varphi = \frac{2e\,\phi}{\hbar} = \frac{2\pi\phi}{\phi_0}$$

when fully symmetric.

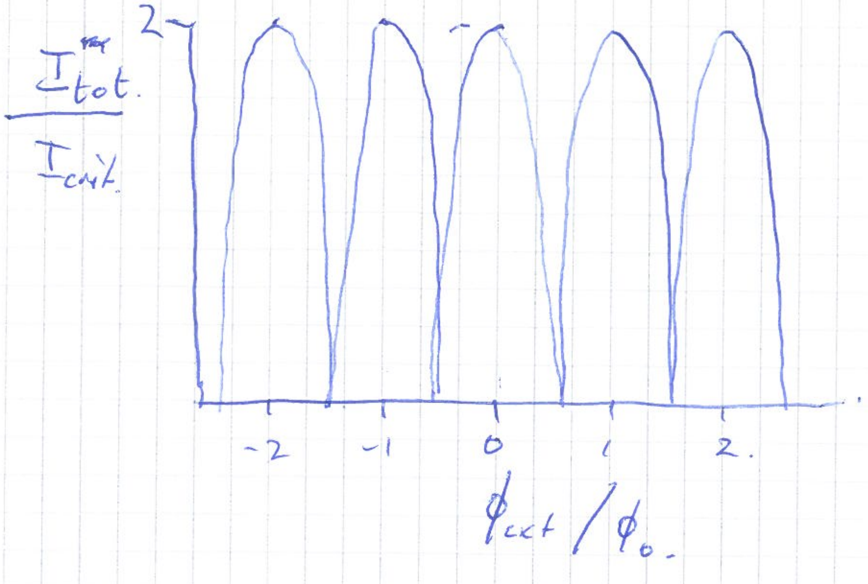
$$I = I_{max}\left(\sin\left(\delta_A + \frac{\pi\phi}{\phi_0}\right) + \sin\left(\delta_B - \frac{\pi\phi}{\phi_0}\right)\right)$$

$$= 2I_{max}\sin\left(\frac{\delta^+}{2}\right)\cos\left(\frac{\delta^-}{2} + \frac{\pi\phi}{\phi_0}\right)$$

$\delta^- = \delta_A - \delta_B$

$\delta^+ = \delta_A + \delta_B$

$\Rightarrow$ ~~max~~ current varies with $\pi\phi/\phi_0$



$\Rightarrow$ extrema for $\frac{\phi_{ext}}{\phi_0} = N$.

technically one can resolve $\Delta\phi_{ext} \simeq 10^{-6}\phi_0$.

for $1\,mm^2$ squid $\Rightarrow 10^{-15}\,T$. $(10^{-6}\phi_0/1mm^2)$

$\phi_0 = 2 \cdot 10^{-15} T/m^2$