Another modern technique to determine the optical constants of samples is ellipsometry. The changes in polarization are measured when a light beam is reflected from or transmitted through a sample. For reflected light, this change in polarization, $S$, is often described with two values $\tan \theta$ and $\Delta$:

$$S = \tan \theta \ e^{-i\Delta} = \frac{R_p}{R_s}$$

where $\theta$ is the ratio of the reflected amplitudes and $\Delta$ is the phase difference produced upon reflection. Polarization changes arise from the reflectivity difference between electric field components oriented parallel ($p$-) and perpendicular ($s$-) to the plane of incidence.

In general, light reflection produces a change in amplitude ratio and phase difference between the $p$- and $s$- components. Ellipsometry measures this change in polarization in order to determine the sample properties.
For the case of metals and semiconductors, the optical properties are heavily influenced by the presence of free electrons. In principle, these can be treated as Lorentz oscillators with \( \omega_0 = 0 \). In the following, we will look in detail at the consequences from this model. We can treat metals (and doped semiconductors) as plasmas because they contain equal numbers of fixed positive ions and free electrons. The free electrons experience no restoring forces when they interact with electromagnetic waves. The model we will discuss here combines the Drude model of free electron conductivity with the Lorentz model of dipole oscillators, and is therefore known as the Drude-Lorentz model. In analogy to the treatment above, we write:

\[
 m_0 \frac{d^2 x}{dt^2} + m_0 \frac{dx}{dt} + \omega_0^2 x = -eE_0 e^{-i\omega t}
\]

where we only leave out the term for the restoring forces. With the annihilation:

\[
 X = X_0 e^{-i\omega t}
\]

we obtain:

\[
 X = \frac{eE}{m_0 (\omega^2 + i\gamma \omega)}
\]

and, as before:

\[
 D = \varepsilon_\infty \varepsilon_0 E
 = \varepsilon_0 E + P
 = \varepsilon_0 E - \frac{Ne^2}{m_0} E
 = \varepsilon_0 E - \frac{Ne^2}{m_0 (\omega^2 + i\gamma \omega)} E
\]
Finally: \( \varepsilon_r = 1 - \frac{N_p^2}{\varepsilon_0 \omega^2} \frac{1}{(\omega^2 + \imath \omega)} \)

\( = 1 - \frac{\omega_p^2}{(\omega^2 + \imath \omega)} \) \hspace{1cm} \text{with} \ \omega_p = \sqrt{\frac{N_p^2}{\varepsilon_0 \omega_0}} \text{ Plasma frequency}

This is again the same result as above only with \( \omega_0 = 0 \).

For the case of weak damping (\( \gamma = 0 \)) we have: \( \varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \)

We get the reflectivity via \( \tilde{m} = \sqrt{\varepsilon_r} \)

\( \tilde{m} \) is imaginary for \( \omega < \omega_p \) and real for \( \omega > \omega_p \), with \( \tilde{m} = 0 \) for \( \omega = \omega_p \). The reflectivity is \( R = \left| \frac{\tilde{m} - 1}{\tilde{m} + 1} \right|^2 \).

For \( \omega < \omega_p \) we have:

\[ \varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2} = \frac{\varepsilon_0^2 - \omega_p^2}{\varepsilon_0^2} \Rightarrow R = \left| \frac{\varepsilon_0^2 - \omega_p^2}{\varepsilon_0^2 + \omega_p^2} \right|^2 = 1 \]

For \( \omega = \omega_p \) we have:

\[ \varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2} = 0 \]

\[ R = \left| \frac{0 - 1}{0 + 1} \right|^2 = 1 \]

For \( \omega > \omega_p \) we have:

\[ \varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2} = \frac{\omega^2 - \omega_p^2}{\omega_p^2} \Rightarrow 1 \]
$$R = \left| \frac{\eta_1 - \eta}{\eta_1 + \eta} \right|^2 = 0$$

This frequency dependence is shown below:

A metal is then completely reflecting up to the plasma edge. As this is usually beyond the visible part of the spectrum, metals appear shiny.
We will now explicitly include damping in our model. Therefore, we rewrite the equation of motion, this time using the electron velocity \( v \):

\[
 m_0 \frac{d v}{d t} + m \frac{d \vec{v}}{d t} = -e \vec{E}
\]

For the case of free electrons, it is often more intuitive to think of the damping time \( \tau \) instead of the damping rate. Define

\[
 m_0 \frac{d v}{d t} + m_0 \cdot \frac{1}{\tau} \cdot v = -e \vec{E}
\]

The solution for \( v \) must have a similar form as the solution for \( x \), with the ansatz \( v = v_0 \, e^{-\omega t} \), one obtains:

\[
 v(t) = -\frac{e\vec{E}}{m_0} \frac{1}{1 - i\omega \tau} \quad \text{E(t)}
\]

For the case of free electrons, one can also use the current and the conductivity to describe the behaviour of the electrons. We have Ohm's law:

\[
 \vec{J} = -N e \vec{v} = \sigma \vec{E}
\]

Current

\[
 \Rightarrow \quad \vec{J} = \frac{N e^2 v}{m_0} \frac{1}{1 - i\omega \tau} \quad \text{E(t)}
\]

\( \sigma(\omega) \) AC conductivity (optical conductivity)

\[
 \sigma(\omega) = \sigma_0 \cdot \frac{1}{1 - i\omega \tau}
\]

with \( \sigma_0 = \frac{N e^2}{m_0} \)
The optical conductivity and the dielectric function are related to each other by

\[ \varepsilon_r(\omega) = 1 + \frac{i \sigma(\omega)}{\varepsilon_0 \omega} \]

Roughly speaking, \( \varepsilon_r \) is used for high frequencies and \( \sigma \) for lower frequencies where the relation to the DC conductivity \( \sigma_0 \) becomes very obvious.

To further understand the optical properties of metals, let us look at some examples.

The plasma frequency depends on the number density of free electrons:

[Equation]

The slide shows a curve representing selected values for \( \omega_p \), \( \omega_0 \), and \( \omega_0/\omega_p \).

It is apparent that very large values of \( N \) lead to plasma frequencies in the ultraviolet spectral region. Above this frequency, metals become transparent. An example is aluminium.

An important mechanism of absorption in metals are interband transitions. Here, a photon moves an electron from an occupied to an unoccupied band. Let us discuss this for our example of Al. The slide shows the band diagram (we have already covered this before. When you look closely, you can see the band folded for electron bands). The absorption rate is proportional to the density of states in the final state.
The band has a gap between the second and the parallel bands. This gap is of the order of 30-50 meV. The band is the same as the left edge of the parallel band. This gap is due to the spin-orbit coupling. The gap is caused by the spin-orbit coupling and the parallel band is, in fact, not parallel. This is due to the spin-orbit coupling. The effect of spin-orbit coupling is also due to the spin-orbit coupling. The gap is due to the spin-orbit coupling. This is due to the spin-orbit coupling.
The onset of the intraband transition about 2 eV explains the red color of copper. The same argument explains the yellow color of gold.

Now, we consider the behavior of the free electrons right at the plasma frequency. Remember, that a plasma consists of a gas of charged particles in dynamic equilibrium. The particles are in constant motion, and this can create local charge fluctuations. When such a fluctuation a small region with an excess charge is created, a force is acting on these electrons with the goal of reaching a homogeneous distribution again. The velocity acquired by the electrons can lead to overshooting and in consequence to an oscillatory motion, the plasma oscillation which have their characteristic frequency right at the plasma frequency. The corresponding quasi-particle is called plasma.

As the plasma is a longitudinal wave, it cannot couple with photons, which are described by a transversal wave. In the next book we will see that they can be detected by inelastic scattering of photons instead. This situation changes for a special plasma mode localized at the surface.
of a metal, the surface plasmon. The associated wave propagates along the interface. The amplitude of the electric field decays exponentially on either side of the interface. It is obvious that the surface plasmon contains also transverse components of the electric field that allow coupling to photons. In the case of strong coupling it is appropriate to speak of a polariton which is a coupled electric polarization–photon wave. The most clear manifestation of surface plasmon effects is found in the optical properties of metal nanoparticles. The color of a suspension of such clusters is dominated by the plasmon resonance. This is exploited since antiquity to produce colored windows by embedding metal nano-particles.