Lecture Notes

Introduction to Strongly Correlated Electron Systems

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# Introduction to strongly correlated electron systems

## I. Introduction

Brief summary of electrons in solids, origin of strong electron correlations

## II. Classes of strongly correlated electron systems

(a) **Transition metal compounds: 3d-electrons**
- Hubbard model, Mott insulator, metal-insulator transition
- Spin, charge, and orbital degrees of freedom and ordering phenomena, selected materials
- Pressure effect on the ground state properties of transition metal compounds

(b) **Heavy fermion systems: 4f (5f) – electrons**
- Landau Fermi-liquid model, Kondo effect, heavy fermion systems, non-Fermi liquid, quantum phase transitions, selected materials
- Pressure effect on the ground state properties of heavy fermion compounds

(c) **Nanoscale structures:**
- Quantum confinement, unusual properties for potential applications

## III. Summary and open discussion
Heavy fermion metallic systems

Intermetallic Ce (4f), Yb (4f) and U(5f) - compounds

increasing hybridization between localized states and conduction electrons

unusual ground states:
Fermi-liquid, Non-Fermi-liquid, heavy fermions, new magnetic and unconventional superconducting states
Physical picture: crossover magnetic ↔ nonmagnetic

Interaction between the Spins of conduction electrons with impurity spins
⇒ Spin correlations

Strong resonace scattering of conduction electrons by the local moments

Formation of an (Abrikosov-Suhl) resonance at $E_F$ of width $k_B T_K$

Logarithmic increase of $\rho$ below $T_K$

$T \ll T_K$:

a) impuity magnetic Moment is screened by the Spins of conduction electrons. This leads to formation of a local Singlet state

b) Energy lowering due to formation of a Kondo-state:

$$k_B T_K = D e^{-\frac{1}{|J| N(E_F)}}$$

Crossover:
magnetic ↔ nonmagnetic
weak ↔ strong coupling
Kondo effect in concentrated alloys

Kondo-lattice systems (heavy fermions)
Properties of Kondo-Lattice systems (Heavy Fermions)

Electrical resistivity: deviation from single ion behavior

Periodicity of the lattice

Coherent scattering of conduction electrons on magnetic impurities

Resonance type increase of the density of state at Fermi level.

Formation of an Abrikosov-Suhl resonance at $E_F$

$T \to 0$: 
\[ \rho(T) = \rho_0 + AT^2 \]
(Fermi-liquid state)

$A \propto [D(E_F)^2]$
(electron-electron interaction)
Kondo-lattice systems (heavy fermions)

Lattice of certain $f$-electrons (most Ce, Yb or U) in metallic environment

Ce$^{3+}$: $4f^1$ ($J = 5/2$), Yb$^{3+}$: $4f^{13}$ ($J = 7/2$)

partially filled inner 4f/5f shells $\rightarrow$ localized magnetic moment

CEF splitting $\rightarrow$ effective $S=1/2$

Formation of spin singlet

$T^* \sim 5 - 50$ K
Kondo-lattice systems (heavy fermions)

characteristic temperature $T^*$

$T \gg T^*$: local moment behavior

$T \ll T^*$: nonmagnetic heavy fermion liquid (Fermi liquid ground state)

See board!
Properties of heavy fermion systems

Normal metal

\[ \rho \sim T^5 \]

\[ \rho \sim T \]

30K

\[ \rho_0 \]

\[ \rho \sim T^2 \]

30K

\[ \rho_0 \]

Heavy fermion system

\[ \rho \sim \ln T \]

Characteristic T* so \( T^* \) analog to Tc

\[ T \gg T_B \] local moment behavior

\[ \chi(T) \] Curie-Weiss

\[ \rho(T) \sim -\ln T \] (Kondo)

\[ T \ll T^* \] nonmagnetic heavy fermion liquid

\[ \Rightarrow \] FL ground state with large \( D(E_F) \)

\[ \chi(T) \propto m^* \propto \frac{1}{T} \]

\[ \gamma(T), \gamma_0 \propto m^* \propto \frac{1}{T^2} \]

\[ \rho(T) \propto \frac{1}{e_e^2} \] (e-e)

\[ \rho(T) \propto AT^2, A \sim \gamma_0^2 \]

Wilson's ratio \( R \approx 1 \)

Show some examples.
Development of coherent scattering in Ce-based alloy

Onuki and Komatsubara (1987)
Kondo-Lattice, heavy fermion systems

Electrical resistivity

Coherent heavy fermions

After Fisk, Ott, Rice & Smith 86
Disappearance of the local moments at low temperatures

\[ \chi (\text{emu/mol}) \]

\[ T (\text{K}) \]

Ce\(_x\)La\(_{1-x}\)Cu\(_6\)

\[ B \parallel c \]

Onuki and Komatsubara (1987)
magnetic susceptibility

After Fisk, Ott, Rice & Smith 86
$C/T = \gamma \ vs \ T^2$ for CeCu$_2$Si$_2$, UBe$_{13}$, and UPt$_3$ very high $\gamma$ (effective mass!)

$\gamma(T)T \approx 1 \text{ J/mol-K}^2$
Wilson's Ratio: $\gamma / T \approx \text{constant}$

$\gamma_{\text{CeCu}_6} \sim 1000 \text{ mJ/mol/K}^2$

$\gamma_{\text{Cu}} \sim 1 \text{ mJ/mol/K}^2$
electron-electron scattering

Kadowaki-Woods plot (1986)

$$\frac{A}{\gamma}$$ is constant and material-independent

Observed for a large number of heavy fermion systems
Theoretical description of heavy fermions
Kondo-lattice model

generalizing the single impurity Anderson model
to a lattice of localized f orbitals hybridizing with conduction band  →  Anderson lattice model
The Anderson model (961)

Hybridization between impurity level and conduction band

\[ \Rightarrow \text{magnetic impurity } d \ (\text{or } f) \text{ level exhibits finite life time } \Rightarrow \text{finite width } \Delta \]

\[ \Rightarrow \Delta = \pi V_k^2 D(E_F) \]

Conduction electrons \( d \) electrons (localized)

\[ H_A = \sum_{k\sigma} \varepsilon(k) c_{k\sigma}^{\dagger} c_{k\sigma} + \sum_{\sigma} \varepsilon_d c_{d\sigma}^{\dagger} c_{d\sigma} + U n_{d\uparrow} n_{d\downarrow} + \]

\[ \sum_{k\sigma} V_k c_{k\sigma}^{\dagger} c_{d\sigma} + V_k^* c_{d\sigma}^{\dagger} c_{k\sigma} \]

Hybridization between local \( d \) electrons and conduction electrons \( (\Delta) \)

Coulomb repulsion between \( d \) electrons
Theoretical description of heavy fermions

Kondo-lattice model: generalizing the single impurity Anderson model (periodic Anderson model) to a lattice of localized f orbitals $f_i$

$H = \sum_{k,\sigma} \epsilon_k c_k^\dagger c_k + \epsilon_f \sum_i f_i^\dagger f_i + U \sum_i n_i^f n_i^{f\sigma} + V \sum_{i,\sigma} (f_i^\dagger c_i^{\sigma} + h.c.)$

- dispersive band, conduction electron flat f-level
- Coulomb repulsion between f-electrons
- hybridization between f and conduction band.

Local f moments if: i) $U$ large and $\epsilon_f$ is negative

Using Schwinger-Wolf transformation $\Rightarrow$ Kondo Lattice model:

$H = \sum_{k,\sigma} \epsilon_k c_k^\dagger c_k + J \sum_i \vec{s}_i \cdot \vec{s}_i$

$J = 2V^2 \left( \frac{1}{\epsilon_f^2} + \frac{1}{\epsilon_f^2 + U} \right) > 0$

Kondo coupling.
Peierls' Anderson model (Kondo lattice model)

Effect of hybridization:
- $f$-level splits due to hybridization with conduction bands
- Formation of two quasiparticle bands

Conduction electrons

Energy

$k$

$V = 0$

$\varepsilon(k)$

$\varepsilon_f$

$V \neq 0$

Conduction electron-like

$f$-electron-like near $E_f$
Consequences of hybridization between f and conduction band

\[ E_F \]

\[ \varepsilon_f \]

\[ \varepsilon(k) \]

\[ V=0 \]

\[ V \neq 0 \]

* A FL ground state can be expected if the local moments are screened by a lattice generalization of the Kondo effect. The onset of Kondo screening at \( T \approx T_K \)

* The resulting FL is formed below a coherence temperature \( T_{coh} \), \( T_{coh} < T_K \)

  \[ \Rightarrow \] Coherent scattering of conduction electron by the local f moments

  \[ \Rightarrow \] Resonance-type increase of the density of states at \( E_F \)

  \[ \Rightarrow \] Formation of an Abrikosov–Suhl resonance at \( E_F \) with width \( \sim k_B T_K \)

  \[ \Rightarrow \] High density of states \( \Rightarrow \) high effective mass \( m^* \) !
Kondo insulators

Simple picture:

- conduction band
- $f$-electrons

$V=0$

$\Delta = \text{hybridization gap}$

$V \neq 0$

$\Delta$

Examples:

- $\text{SmB}_6$
- UNiSn

(see high pressure examples)
Consequences of hybridization:

⇒ f-electrons participate to the Fermi surface (FS)
⇒ large volume of the Fermi surface

\[
\frac{V_{FS}}{\bar{V}} = 4\pi \frac{3}{4} (\bar{n}_c + \bar{n}_f) \quad , \quad \bar{n}_c = \bar{n}_c \quad \text{of conduction elec.} \quad \bar{n}_f = -\bar{n}_f \quad \text{f-electrons}
\]

Further comments:

* \( T > T_{coh} \) ⇒ local moments exist ⇒ system has a Fermi volume described by conduction electrons ⇒ \( \frac{V_{FS}}{\bar{V}} = \frac{4\pi}{3} \bar{n}_c \) (usually small)

* \( T \sim T_{coh} \) : The FS fluctuates strongly ⇒ resistivity is enhanced

\( T_{coh} \leftrightarrow \text{resistivity maximum (experimental point of view)} \)

* \( T < T_{coh} \) : ⇒ Screening of local moments:

- FL behavior in the Kondo lattice competes with interaction between the local moments (indirect RKKY interaction) ⇒ long range magnetic order
- other competitors of the FL maybe magnetic frustration, spin glasses and spin liquid
Experimental support for description of heavy fermion metals by Landau Fermi Liquid Theory

- The volume of the Fermi surface includes the \( f \) electrons.

- The measured quasiparticle mass accounts for the enhanced specific heat.

Both these observations confirm the success of Fermi liquid theory.
Momentum distribution

\[ \langle n(k) \rangle \]

mostly c-character

(Small) \[ F \]

mostly f-character

\( k_{(large)} \)

\( k \)

\( \downarrow Z \)
The $f$ electrons participate to the Fermi surface

Example: UPt3

Figure 7: The consistency of the Fermi liquid description has been demonstrated in UPt3. The Fermi surface sheets (from Julian and McMullan 1998) and Effective mass of the quasiparticles have been mapped out by de Haas van Alphen measurements (Taillefer and Lonzarich 1988). They confirm that the 5$f_3$ electrons are absorbed into the Fermi liquid and that the quasiparticle masses are consistent with the mass enhancements measured in specific heat (after Stewart et al. 1984). The percentages reflect the contribution from quasiparticles on each sheet to the total specific heat.
can we observe the Kondo resonance experimentally?

**Kondo resonance**

Formation of an (Abrikosov-Suhl) resonance at $E_F$ of width $k_B T_K$
High-resolution photoemission spectroscopy of CeCu2Si2

Reiner et al PRL (2001)

Kondo resonance peak at $E_F$
Theoretical prediction

Evolution of the Kondo resonance peak at $E_F$
at low temperatures

Abb. 8: Hochaufgelöstes Photoemissionsspektrum an CeCu$_2$Si$_2$ bei $T = 11$ K in der Nähe der Fermi-Energie, vor und nach einer Normierung auf die Fermi-Verteilung. Nach der Normierung wird die Existenz der scharfen Kondo-Resonanz deutlich, deren Breite und Energie nur wenige Millielektronenvolt beträgt.
Kondo Effect in Mesoscopic Systems
Nanotechnology has rekindled interest in the Kondo effect, one of the most widely studied phenomena in condensed-matter physics.

Revival of the Kondo effect
Leo Kouwenhoven and Leonid Glazman,

WHY would anyone still want to study a physical phenomenon that was discovered in the 1930s, explained in the 1950s and has been the subject of numerous reviews since the 1970s? Although the Kondo effect is a well-known and well-studied phenomenon in condensed-matter physics, it continues to captivate the imagination of experimentalists and theorists alike.

The effect arises from the interaction between a single magnetic atom, such as cobalt, and the many electrons in an otherwise non-magnetic metal. Such an impurity typically has an intrinsic angular momentum or “spin” that interacts with the electrons. As a result, the mathematical description of the system is given as a single entity. Indeed, superconductivity is a prime example of a y-electron phenomenon.

Her metals, like copper and gold, in conducting and have a constant resistance, even at the lowest achievable temperatures. The value of the temperature resistance depends on the number of defects in the material. Increasing the number of defects increases the value of the “saturation resistance” but the characteristic temperature dependence is the same.

However, this behaviour changes dramatically when magnetic atoms, such as iron, are added. Rather than saturating, the electrical resistance increases as the temperature is lowered, further...
Quantum dots – mesoscopically fabricated, tunneling of single electrons from contact reservoir controlled by gate voltage

Regions that can hold a few hundred electrons! Can drive a current through these! This is Nano!

How is conduction related to the Kondo effect?
1. The Kondo effect in metals and in quantum dots

(a) As the temperature of a metal is lowered, its resistance decreases until it saturates at some residual value (blue). Some metals become superconducting at a critical temperature (green). However, in metals that contain a small fraction of magnetic impurities, such as cobalt in copper systems, the resistance increases at low temperatures due to the Kondo effect (red).

(b) A system that has a localized spin embedded between metal leads can be created artificially in a semiconductor quantum-dot device containing a controllable number of electrons. If the number of electrons confined in the dot is odd, then the conductance measured between the two leads increases due to the Kondo effect at low temperature (red). In contrast, the Kondo effect does not occur when the dot contains an even number of electrons and the total spin adds up to zero. In this case, the conductance continuously decreases with temperature (blue).
(a) The Anderson model of a magnetic impurity assumes that it has just one electron level with energy $\varepsilon_0$ below the Fermi energy of the metal (red). This level is occupied by one spin-up electron (blue). Adding another electron is prohibited by the Coulomb energy, $U$, while it would cost at least $|\varepsilon_0|$ to remove the electron. Being a quantum particle, the spin-up electron may tunnel out of the impurity site to briefly occupy a classically forbidden “virtual state” outside the impurity, and then be replaced by an electron from the metal. This can effectively “flip” the spin of the impurity. (b) Many such events combine to produce the Kondo effect, which leads to the appearance of an extra resonance at the Fermi energy. Since transport properties, such as conductance, are determined by electrons with energies close to the Fermi level, the extra resonance can dramatically change the conductance.
Kondo effect in quantum dot

Single quantum dot

\[ T = 15 \text{ mK} \quad \text{and} \quad T = 800 \text{ mK} \]

conductance anomalies
L. Kouwenhoven *et al.* *science* 289, 2105 (2000)

Glazman *et al.* *Physics world* 2001

Glazman et al. *Physics World* 2001
Quantized conductance vs temperature

Universal relation between dimensionless conductance and temperature!

Gate voltage is used to tune $T_K$; measurements at 50 to 1000 mK.
(a) By manipulating cobalt atoms on a copper surface, Don Eigler and colleagues at IBM have placed a single cobalt atom at the focal point of an ellipse built from other cobalt atoms (bottom). The density of states (top) measured at this focus reveals the Kondo resonance (left peak). However, elliptical confinement also gives rise to a second smaller Kondo resonance at the other focal point (right) even though there is no cobalt atom there. (b) Meanwhile, Mike Crommie and co-workers have measured two Kondo resonances produced by two separate cobalt atoms on a gold surface (top). When two cobalt atoms are moved close together using an STM, the mutual interaction between them causes the Kondo effect to vanish (data not shown).
Question:

What happens if the Kondo lattice system is magnetically ordered?

First step

We consider magnetic interaction between localized 4f moments in a metallic systems!
Local versus Itinerant magnetic moments

4f states are highly localized
No direct interaction possible!

Bandwidth ($W$) of the metallic state

- $W(\text{Fe}) \approx 4 \text{ eV}$
- $W(\text{Pu}) \approx 2 \text{ eV}$
- $W(\text{Sm}) \leq 1 \text{ eV}$
Exchange in Rare Earths (4f electrons)

- **Indirect exchange between 4f moments occurs:**

  This type of exchange was first proposed by Ruderman and Kittel and later extended by Kasuya and Yosida to give the theory now generally know as the **RKKY** interaction.

  It is the dominant exchange interaction in metals where there is little or no direct overlap between neighboring magnetic electrons.
RKKY interaction

Spin density

distance
RKKY interaction

Spin density

distance
RKKY interaction

Spin density

distance
RKKY interaction

Spin density

distance
RKKY interaction
RKKY interaction: Description

Local moments (Spin $S_i$) in a sea of conduction electrons with itinerant spin $s(r)$

$$J(r) = 6\pi \, Z \, J \, N(E_F) \left[ \frac{\sin(2k_F r)}{(2k_F r)^4} - \frac{\cos(2k_F r)}{(2k_F r)^3} \right]$$

$Z$ number of electrons / atom

$J$ s-d exchange interaction

$D(E_F)$ DOS at Fermi energy

$k_F$ Fermi momentum

$r$ distance between impurities

$=>$ Oscillations of value and sign

P.H. Dederichs in: Magnetismus von Festkörpern und Grenzflächen, 24. IFF Ferienkurs, Jülich 1993
Question:

What happens if the Kondo lattice system is magnetically ordered?

We consider in Kondo lattice the relative strength of RKKY interaction and that of the Kondo effect.
Theoretical description

Kondo-lattice-system: periodical arrangement of localized 4f-moments in a metallic matrix

Intersite interaction: RKKY, \( E_{\text{RKKY}} = k_B T_{\text{RKKY}} \)

\[ T_{\text{RKKY}} \sim N(E_F) J^2 \]

J: interaction between f- and conduction electrons

Intrasite (on-site) interaction: Kondo-Effect

\( E_K = k_B T_K \)

\[ T_K \sim \exp(-1/N(E_F)J) \]

⇒ screening of the magnetic moments

⇒ nonmagnetic ground state

⇒ long range magnetic order
The main result ... is that there should be a **second-order transition at zero temperature (at QCP)**, as the exchange coupling $J$ is varied, between an antiferromagnetic ground state for weak $J$ and a Kondo-like state in which the local moments are quenched.
• classical phase transition: driven by thermal fluctuations
Quantum Phase Transitions

- classical phase transition: driven by thermal fluctuations
- quantum phase transition: driven by quantum fluctuations
• classical phase transition: driven by thermal fluctuations
• quantum phase transition: driven by quantum fluctuations
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Types of Phase Transitions

First-order

Second-order
Order parameter of a phase transition

Partial list of transitions and order parameters

<table>
<thead>
<tr>
<th>Transition</th>
<th>Order parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid-gas</td>
<td>density</td>
</tr>
<tr>
<td>Ferromagnetic</td>
<td>magnetization</td>
</tr>
<tr>
<td>Ferroelectric</td>
<td>polarization</td>
</tr>
<tr>
<td>Superconductors</td>
<td>complex gap parameter</td>
</tr>
<tr>
<td>Siperfluid</td>
<td>condensate wave function</td>
</tr>
<tr>
<td>Phase Separation</td>
<td>concentration</td>
</tr>
</tbody>
</table>

The main result ... is that there should be a **second-order transition at zero temperature (at QCP)**, as the exchange coupling is varied, between an antiferromagnetic ground state for weak \( J \) and a Kondo-like state in which the local moments are quenched.
Consequence: The nature of the ground state of the system strongly depends on the relative strength of RKKY interaction and Kondo effect.
Non-Fermi liquid behavior at magnetic quantum phase transitions

General aspects:

NFL behavior has been observed in $U_Ce_Yb$ intermetallic compounds:

- chemically substituted: e.g. $Y_{1-x}U_xPo_3$, $UCu_5Po_4$
  (disordered systems)

- stoichiometric: $(\rho = 0) \Rightarrow UBe_3$, $CeCoIn_5$, $YbRh_2Si_2$
  (proximity to QCP)

  $\rho > 0$ (pressure tuned) $\Rightarrow CeIn_3$, $CePo_2Si_2$, $UGe_2$

\[ \Rightarrow \text{different routes to NFL behavior!} \]

* NFL behavior due to disorder: distribution of the Kondo temperature
  (see Stewart, Rev. Mod. Phys. 73, 797 (2001) (distribution of $J$ and or $D(E_F)$)

* multi-channel Kondo effect:

  f-electron spin is overscreened by the spins of conduction electrons
  $\Rightarrow$ antiferromagnetic superexchange interaction with electrons
  off the impurity site

* proximity to a QCP $\rho \Rightarrow$ induced by quantum fluctuations
**Physical properties**: Weak power law, logarithmic divergence in $T$ at low temperatures ($T \ll T_0$):

- Electric resistivity, $\rho \sim a T^x$ with $1 \leq x \leq 1.6$ non-quadratic

- Specific heat divided by $T$, $C/T \sim \ln(T_0/T)$

- Magnetic susceptibility, $\chi = \chi_0 \sim \ln(T_0/T)$

$\Rightarrow$ diverging

Note that:

The appreciable $T$-dependence below $T_0$:

$\Rightarrow$ lower energy scale than Fermi liquid
few comments to proximity to a QCP:

* At the QCP, the low temperature thermodynamics is determined by collective modes corresponding to fluctuations of the order parameter, rather than by single-fermion excitations as in FL $\Rightarrow$ NFL properties arise.

* NFL can also occur near quantum spin-glass or superconducting transitions.

* A quantum phase transition (like thermal or classical phase transitions) is characterized by a diverging correlation length $\xi$ and a diverging relative time $\tau$. However: (a) the critical fluctuations are quantum fluctuations rather than thermal fluctuations, and (b) contrary to classical critical point, the dynamic and static behavior of a QCP are coupled together.

$\Rightarrow$ a system at a QCP will be affected in the same way by either a finite frequency or a finite temperature.
<table>
<thead>
<tr>
<th>Type</th>
<th>Material</th>
<th>( T^* )</th>
<th>( T_c, x_c, B_c )</th>
<th>Properties</th>
<th>( \rho )</th>
<th>( \gamma_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal</td>
<td>CeCu6</td>
<td>10K</td>
<td>-</td>
<td>Simple HF Metal</td>
<td>( T^2 )</td>
<td>1600</td>
</tr>
<tr>
<td>Superconductors</td>
<td>CeCu2Si2</td>
<td>20K</td>
<td>( T_c=0.17K )</td>
<td>First HFSC</td>
<td>( T^2 )</td>
<td>800-1250</td>
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<td></td>
<td>UBe13</td>
<td>2.5K</td>
<td>( T_c=0.86K )</td>
<td>Incoherent metal→HFSC</td>
<td>( \rho_c \sim 150 \mu \Omega \text{cm} )</td>
<td>800</td>
</tr>
<tr>
<td></td>
<td>CeCoIn5</td>
<td>38K</td>
<td>( T_c=2.3K )</td>
<td>Quasi 2D HFSC</td>
<td>( T )</td>
<td>750</td>
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<tr>
<td>Kondo Insulators</td>
<td>Ce3Pt4Bi3</td>
<td>( T_X \sim 80K )</td>
<td>-</td>
<td>Fully Gapped KI</td>
<td>( \sim \epsilon \Delta / T )</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>CeNiSn</td>
<td>( T_X \sim 20K )</td>
<td>-</td>
<td>Nodal KI</td>
<td>Poor Metal</td>
<td>-</td>
</tr>
<tr>
<td>Quantum Critical</td>
<td>CeCu6-xAvx</td>
<td>( T_0 \sim 10K )</td>
<td>( x_c = 0.1 )</td>
<td>Chemically tuned QCP</td>
<td>( T )</td>
<td>( \frac{1}{T_0} \ln \left( \frac{T_0}{T} \right) )</td>
</tr>
<tr>
<td></td>
<td>YbRh2Si2</td>
<td>( T_0 \sim 24K )</td>
<td>( B_{\perp}=0.06T )</td>
<td>Field-tuned QCP</td>
<td>( T )</td>
<td>( \frac{1}{T_0} \ln \left( \frac{T_0}{T} \right) )</td>
</tr>
<tr>
<td>SC + other Order</td>
<td>UPd2Al3</td>
<td>110K</td>
<td>( T_{AF}=14K ), ( T_{sc}=2K )</td>
<td>AFM + HFSC</td>
<td>( T^2 )</td>
<td>210</td>
</tr>
<tr>
<td></td>
<td>URu2Si2</td>
<td>75K</td>
<td>( T_1=17.5K ), ( T_{xc}=1.3K )</td>
<td>Hidden Order &amp; HFSC</td>
<td>( T^2 )</td>
<td>120/65</td>
</tr>
</tbody>
</table>
CeCu$_{6-x}$Au$_x$:

V. Löhneysen et al.:

Quantum critical
Concentration dependence
$C/T \propto \ln(T/T_0)$
$\Rightarrow$ NFL

AF order

$QCP$
($x=0.1$)

Heavy Fermion $C/T = \gamma$
$\Rightarrow$ FL

Pressure dependence

$\rho \propto T \Rightarrow$ NFL

Pressure-induced NFL for $x=0.2$ and $x=0.3$
Coincidence with $x=0.1$ at $p=0$!
CePd$_2$Si$_2$: a low temperature antiferromagnet. Under pressure the antiferromagnetism can be suppressed to zero temperature giving a quantum critical point. Not only non-Fermi liquid behavior but also there is a superconducting transition (after Julian et al. 1996, Mathur et al. 1998).

The resistivity of CePd$_2$Si$_2$ at the critical pressure (28 kbar). The observed temperature dependence, $T^{1.2}$, is seen over two decades of temperature. (Data after Grosche et al. 1996.)
Metallic systems on the border of itinerant electron magnetism

Heavy fermion systems

Rich Phase Diagrams Exhibiting both NFL behavior and superconductivity.

<table>
<thead>
<tr>
<th></th>
<th>(Y_{1-x}U_x\text{Pd (NFL)})</th>
<th>Fermi Liquid</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Heat Capacity</strong></td>
<td>(C \sim -T \ln(T))</td>
<td>(C = \gamma T)</td>
</tr>
<tr>
<td><strong>Conductivity</strong></td>
<td>(\rho \sim \rho_0 + AT^{1.1})</td>
<td>(\rho = \rho_0 + AT^2)</td>
</tr>
<tr>
<td><strong>Magnetic Susceptibility</strong></td>
<td>(\chi_m \sim \alpha - \beta T^{1/2})</td>
<td>(\chi_m = \beta)</td>
</tr>
</tbody>
</table>

Source: Seaman et al.
YbRh2Si2:

Field-induced quantum critical point

NFL behavior @ $B = 0$ in $\beta$-YbAlB$_4$

$B = 0$:
- Non Fermi Liquid
- $\rho_{ab} \propto T^{1.5}$
- $M/H \propto T^{-0.5}$
- $C/T \propto \ln(T^*/T)$

In magnetic field:
- Fermi Liquid
- $\rho_{ab} \propto T^2$
- $M/H = \text{const.}$
- $C/T = \text{const.}$

FL behavior is recovered in magnetic field.

QCP at $B = 0$ under $P = 0$?

Normally, we have to tune $B$ or $P$ or doping to approach QCP.

ex) YbRh$_2$Si$_2$, CeCoIn$_5$, CeCu$_{5.9}$Au$_{0.1}$, …